Lecture 8. LTL Synthesis

ELEC-E8110 Automation Systems Synthesis and Analysis

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LTL synthesis: problem formulation

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- LTL synthesis problem: synthesize a controller such that for all possible behaviors of the environment f is satisfied

LTL synthesis: examples

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- $f = \mathbf{F}(x \wedge y)$

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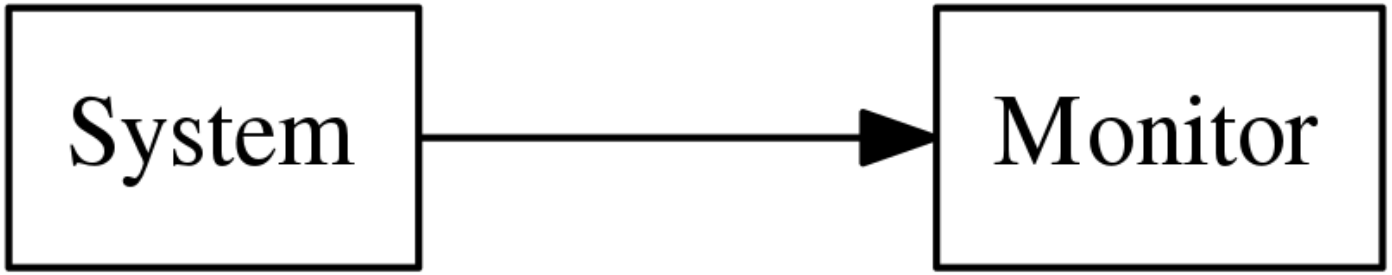
LTL synthesis: encoding the plant

- If the plant is finite-state, it is possible to encode it as an LTL formula
- If f_p describes the plant and f_c are the requirements for the controller assuming that the plant submits to f_p, then it is sufficient to synthesize a controller for $f = f_p \rightarrow f_c$
- The environment still can assign any possible values for inputs, but if it violates f_p, then the controller wins
- Recall: in G4LTL-ST, f_p was specified with ASSUME

To see how the LTL synthesis problem can be solved, we will look into the automata-theoretic approach to LTL model checking

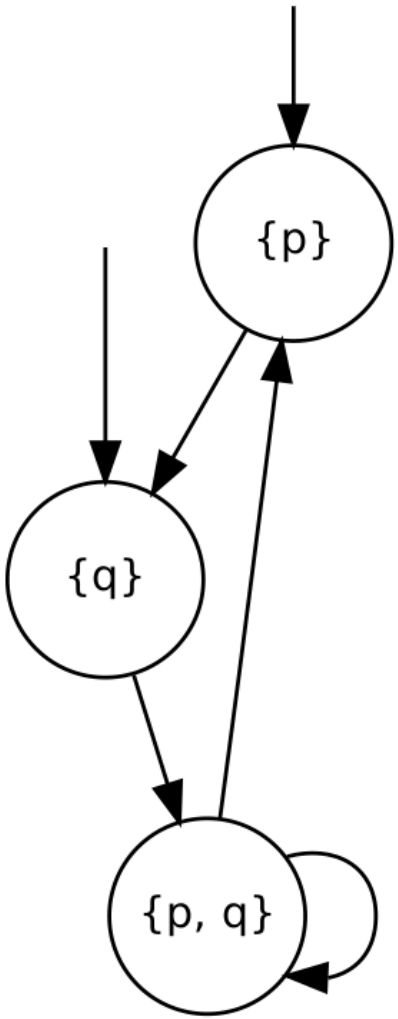
Monitors



- Runtime scenario: can we catch a specification violation while the system (or its model) is operating?
- Assume that we have a Kripke structure of the system, then the monitor has access to atomic propositions on each step
- If we implement the monitor as a state machine, then it can have memory about previous assignments of atomic propositions

Monitors: example of a safety automaton

Assume that we have a Kripke structure...



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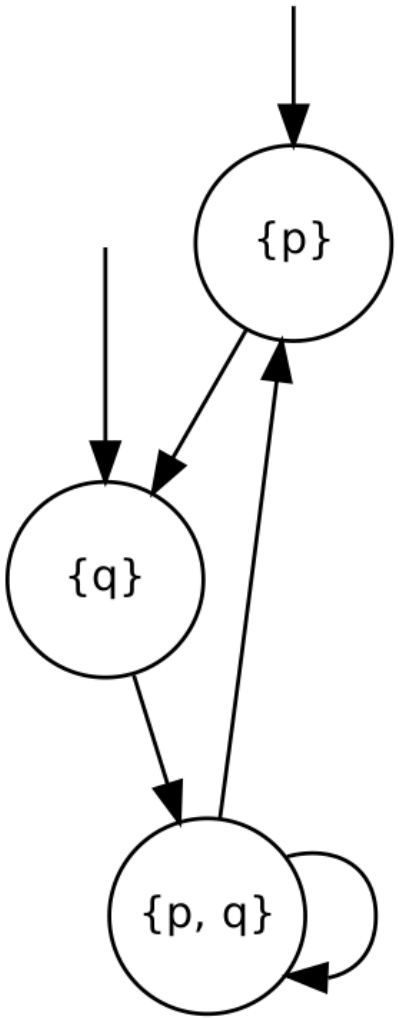

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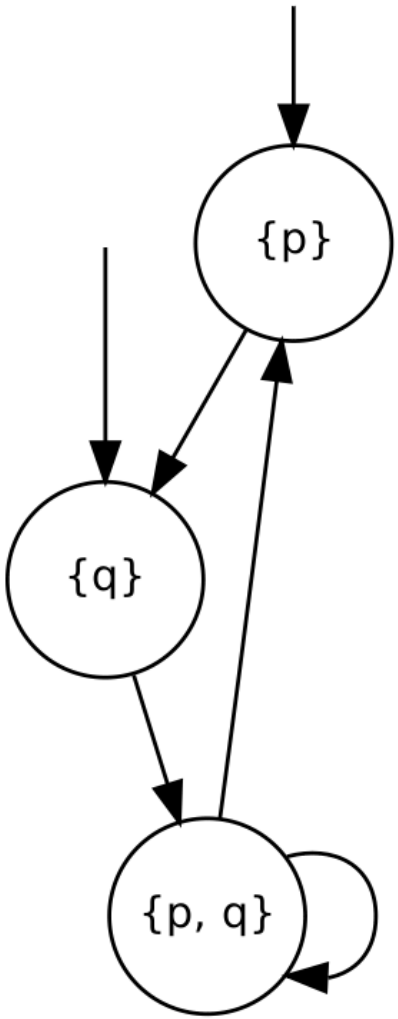

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State machine to check f? With guards on transitions and a rejecting state

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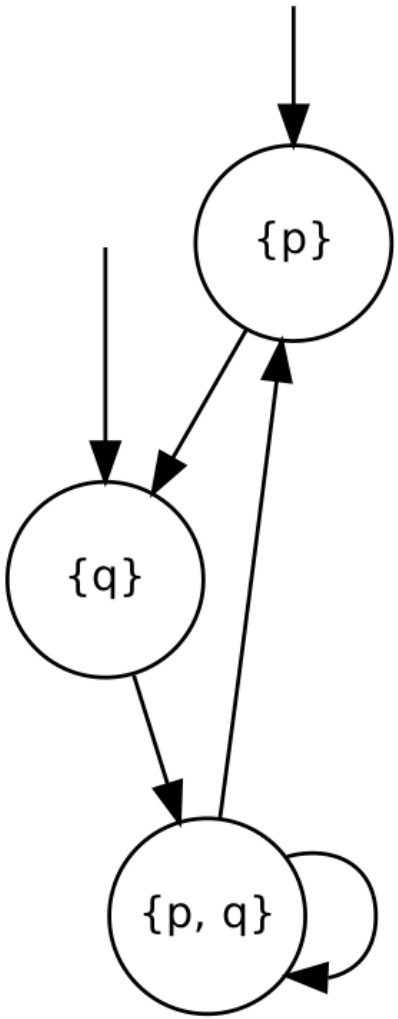

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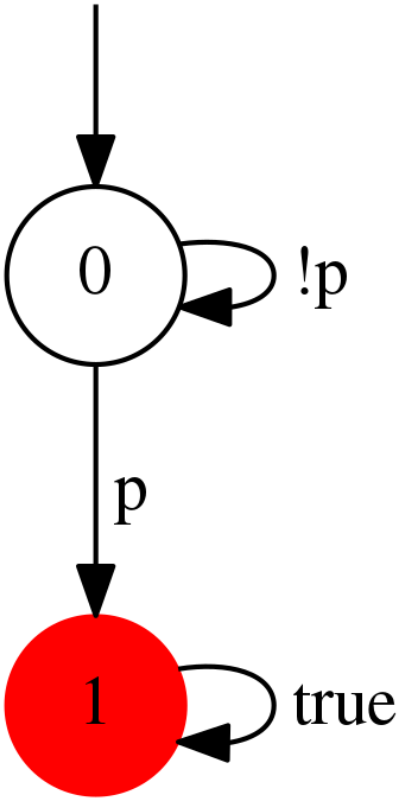
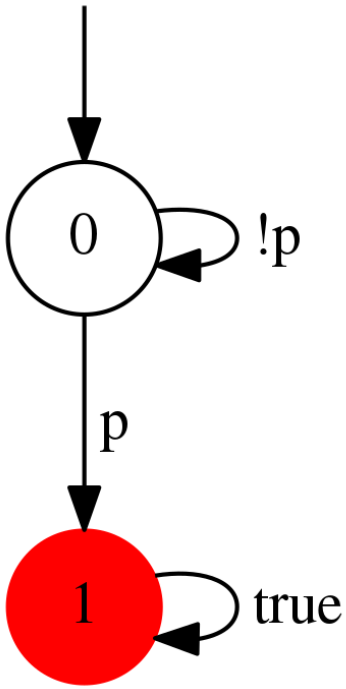


Safety LTL properties and safety automata

- An LTL formula f is a **safety** formula, if all possible counterexamples to f have a **finite prefix** such that every its infinite continuation is a counterexample
- Informally speaking, such properties state that "something bad" never happens
- Each safety property can be converted to a (possibly nondeterministic) safety automaton
- Safety automaton rejects an input sequence if it can visit a rejecting state while reading it

Represent given properties as safety automata (1)

$f_1 = \mathbf{G}(\neg p)$



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$$f_2 = x \wedge \mathbf{X}\, y$$

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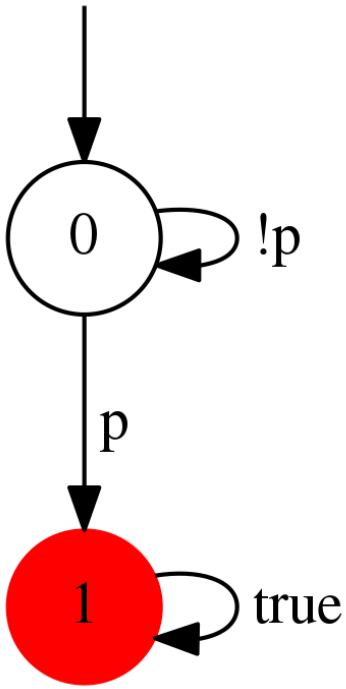

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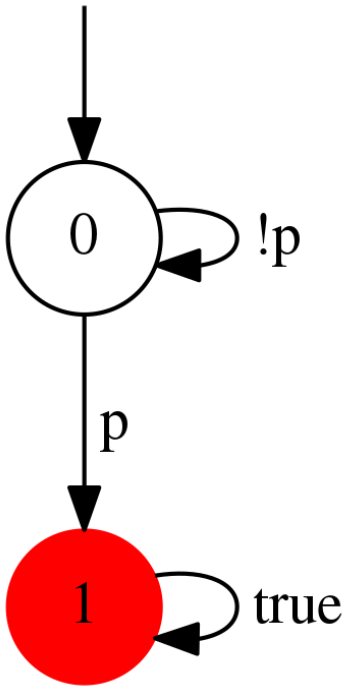

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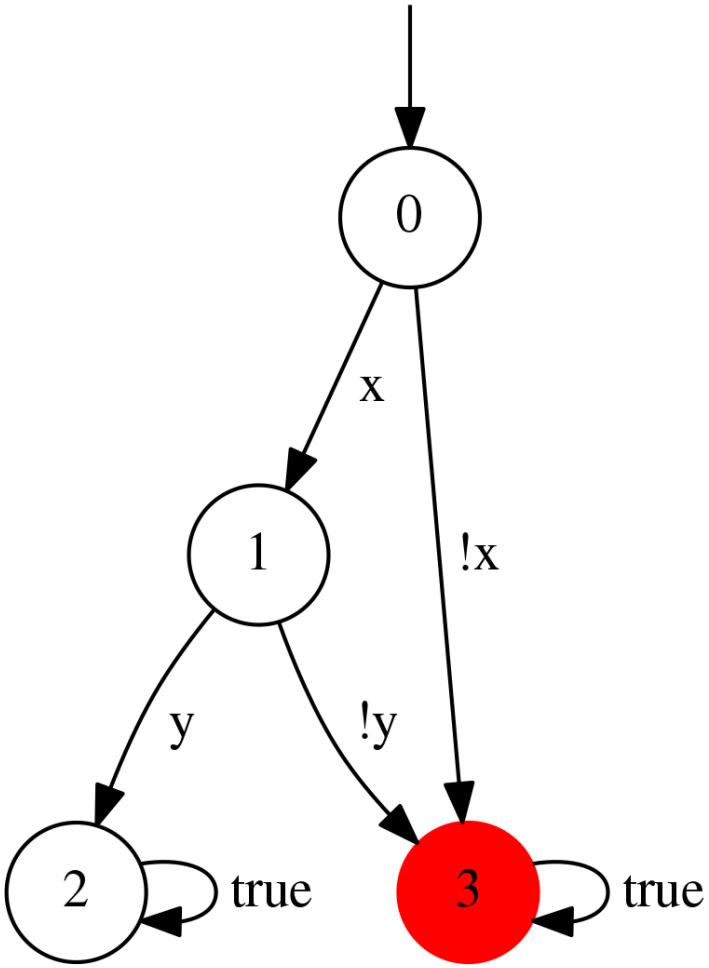

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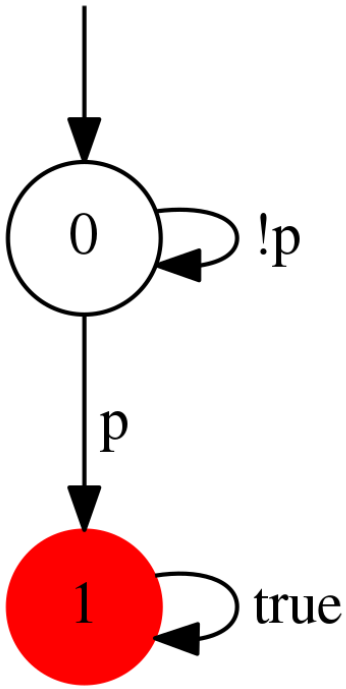

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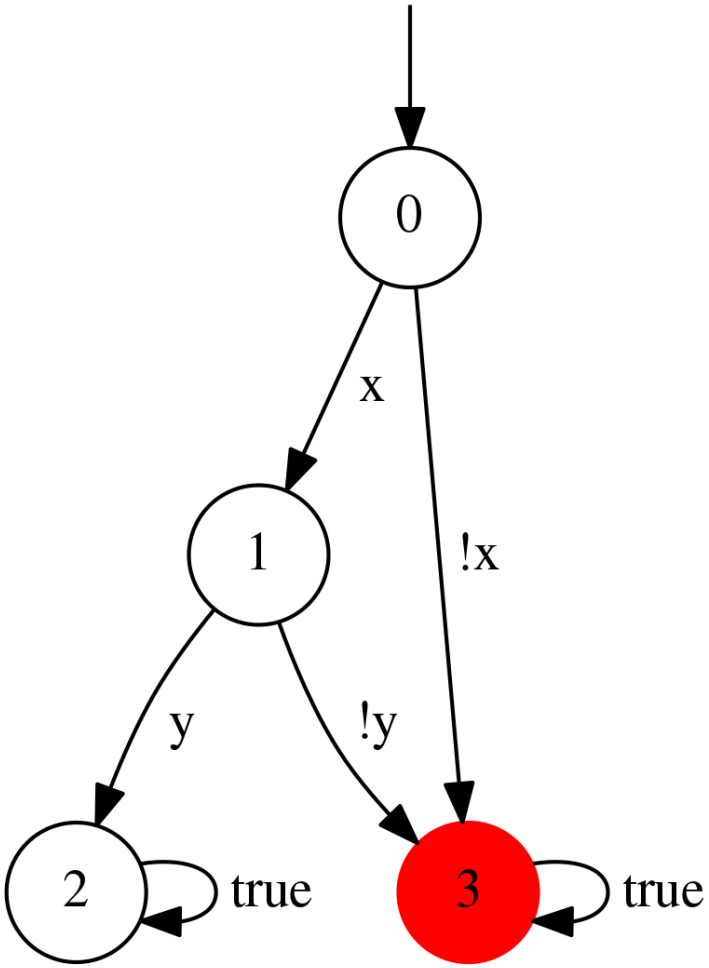

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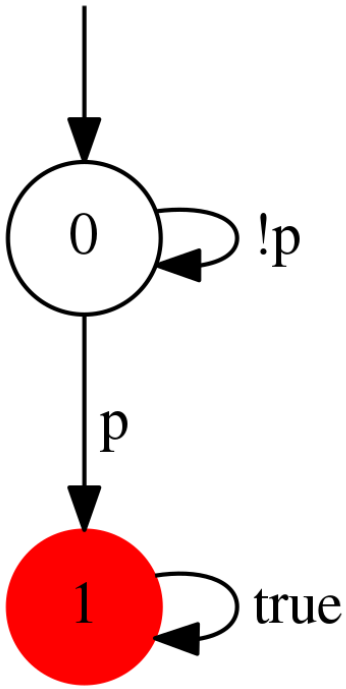

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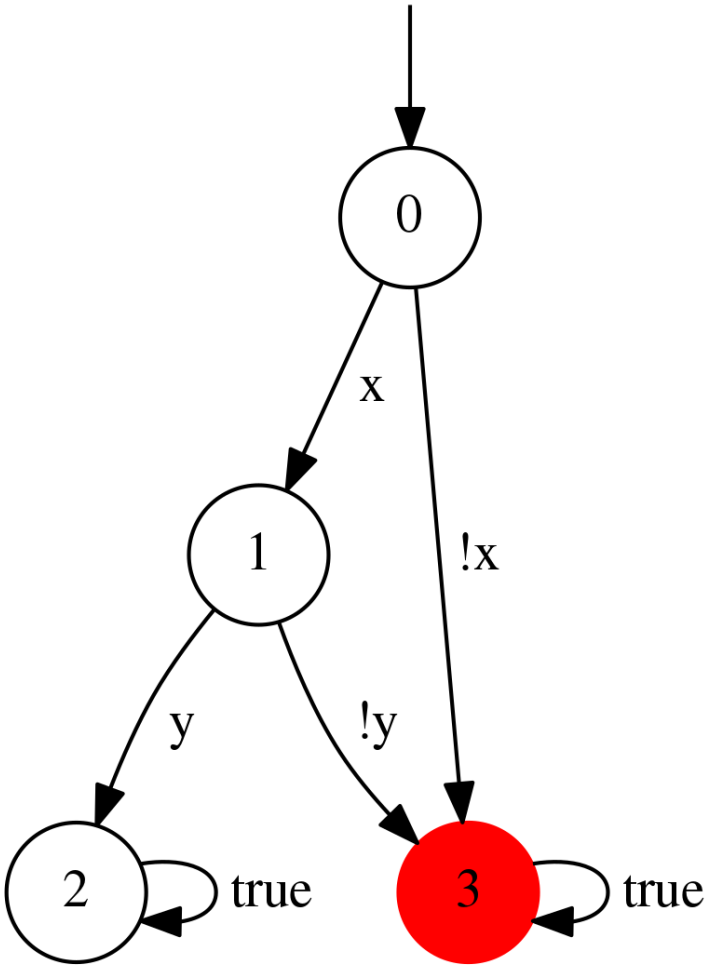

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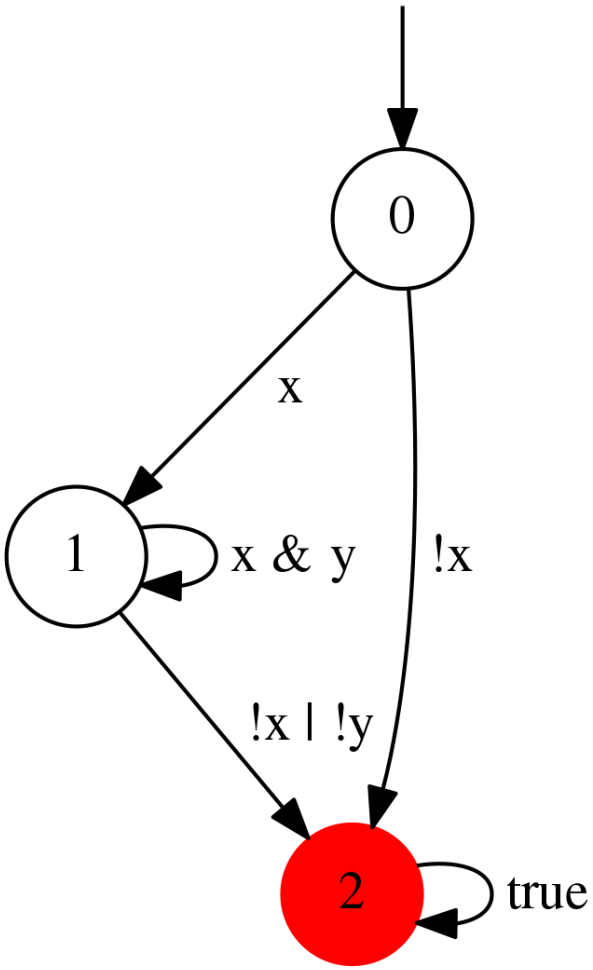

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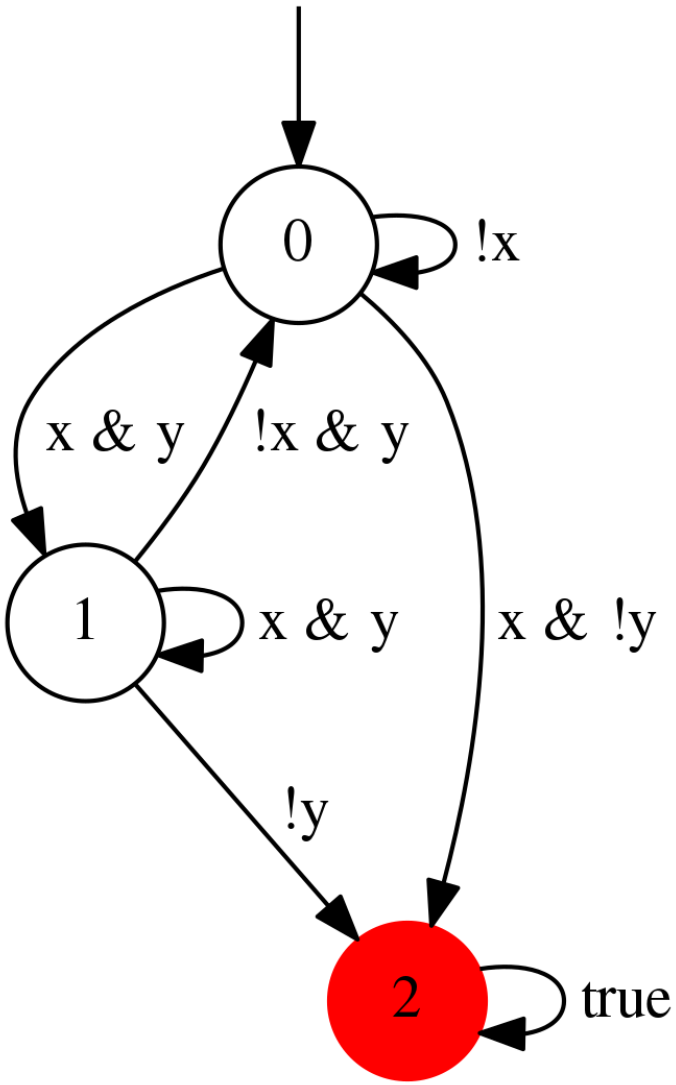

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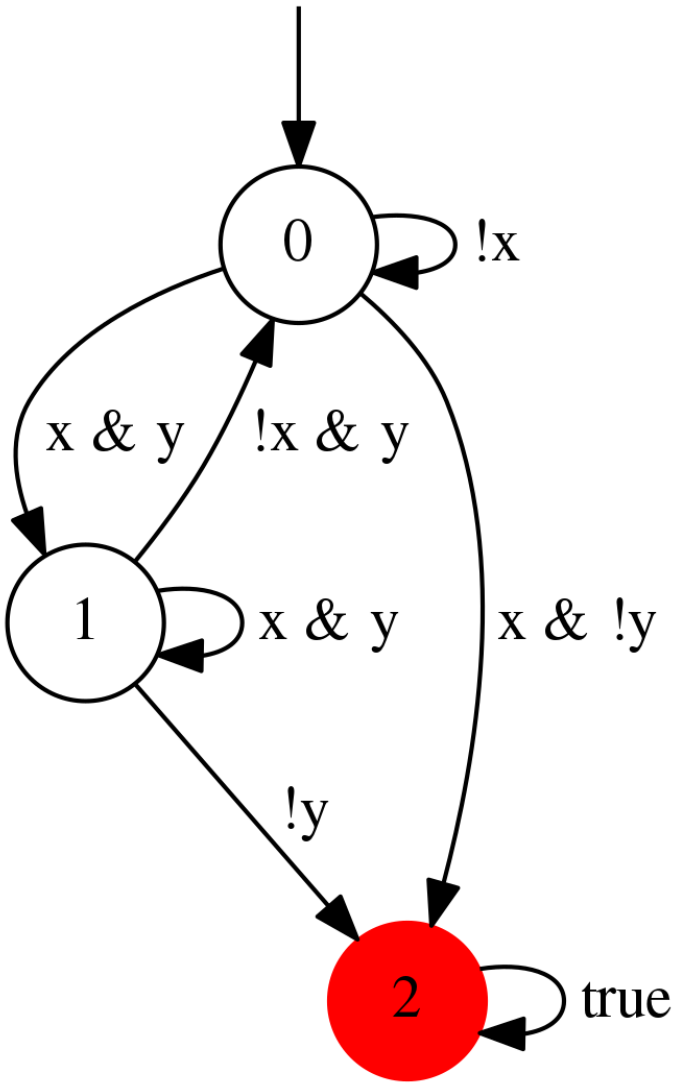

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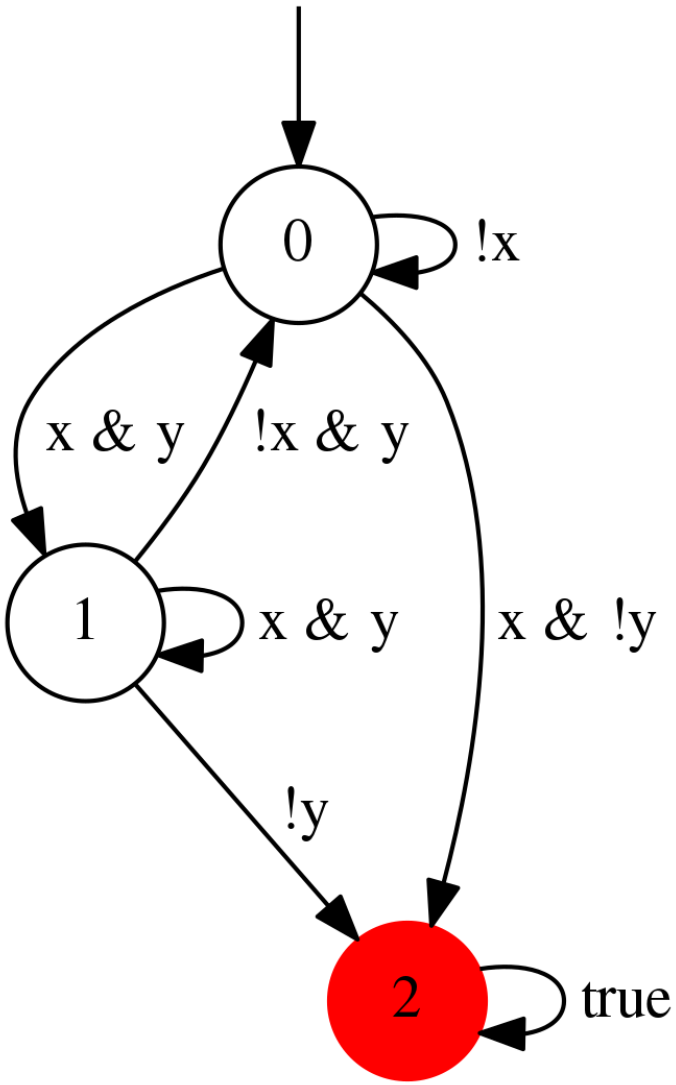

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Not a safety property!

Transforming automata to safety games

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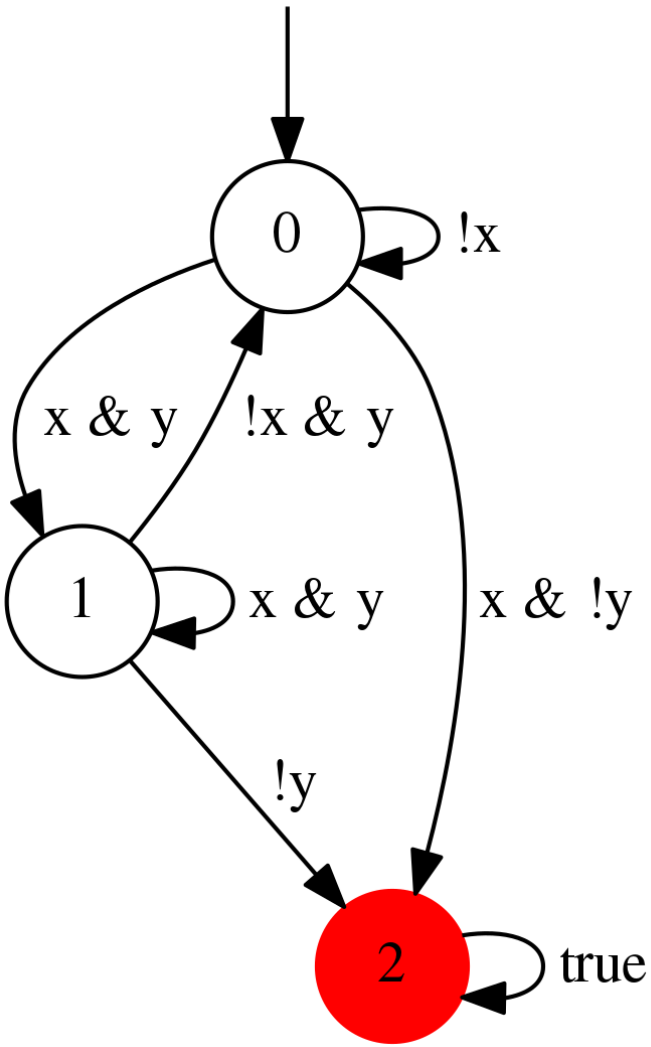

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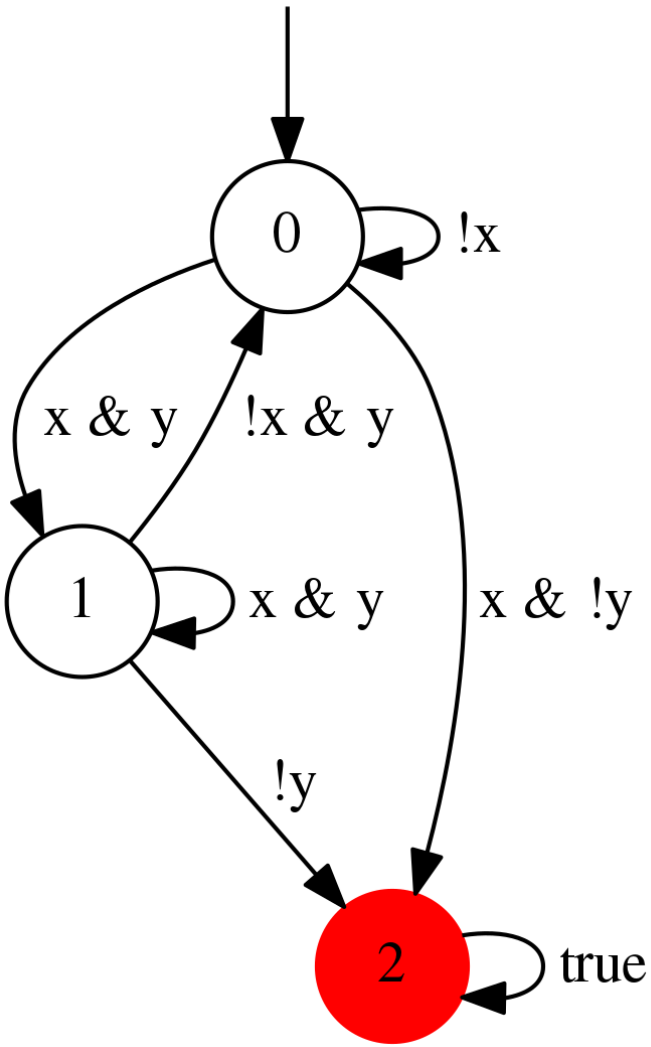

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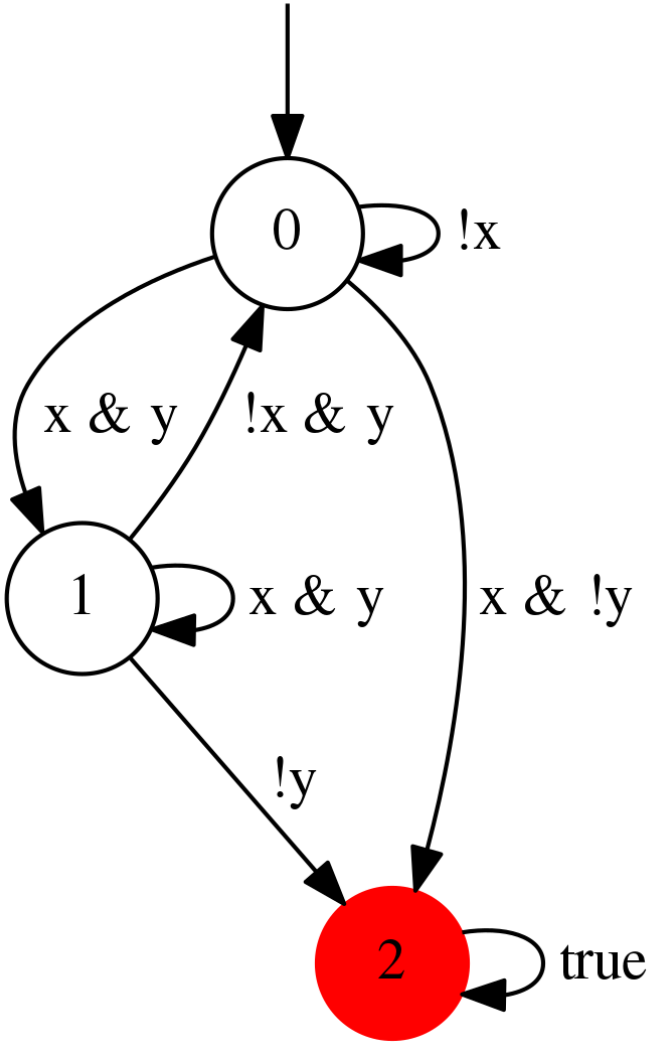

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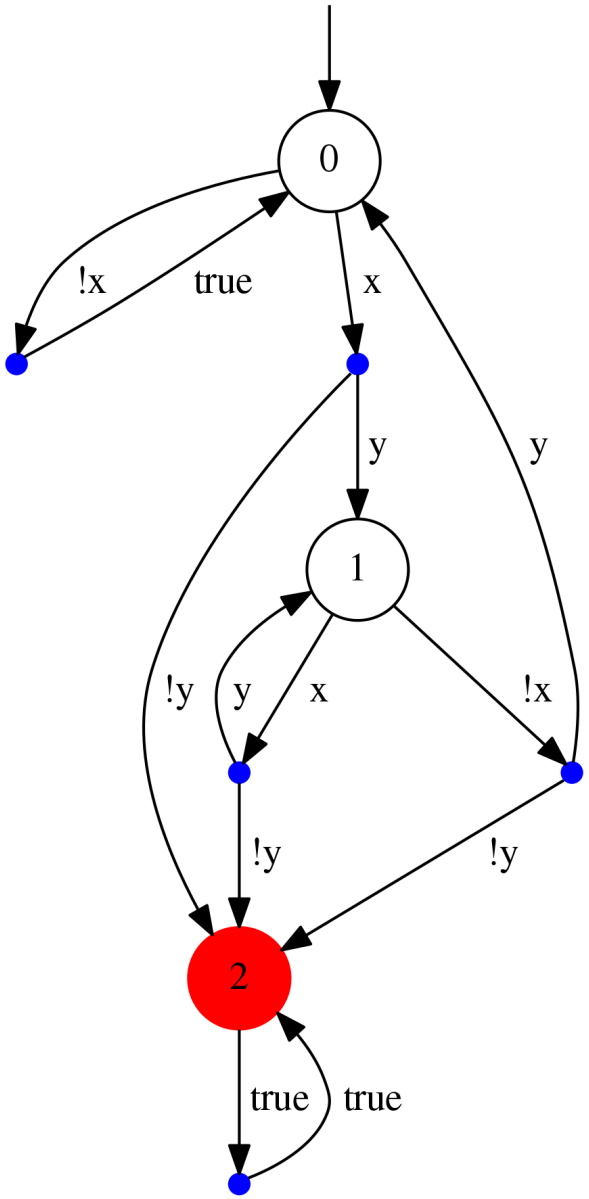

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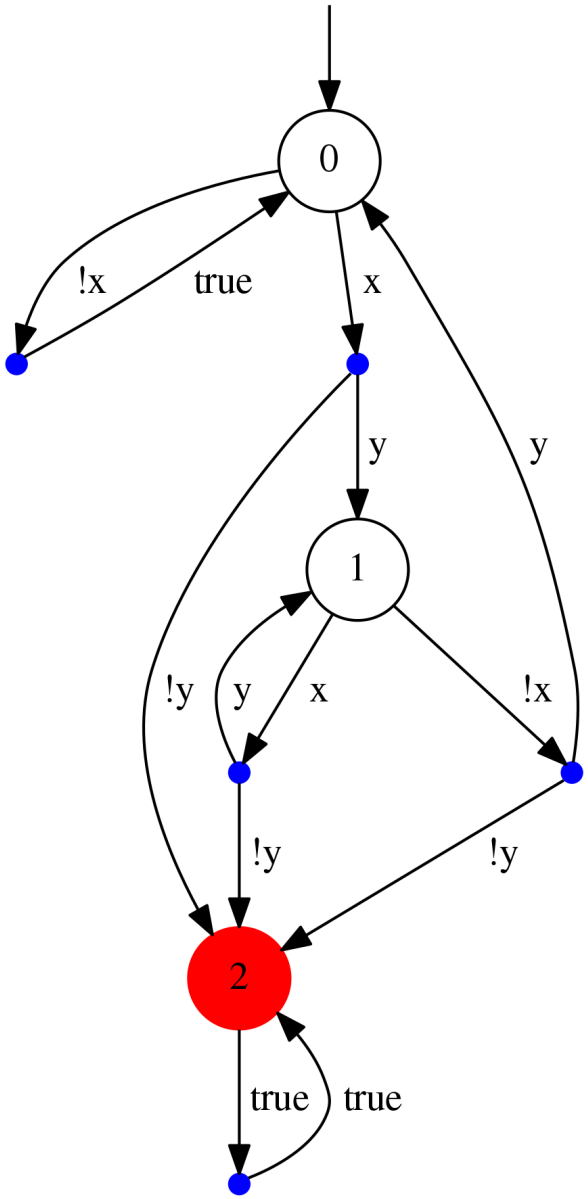

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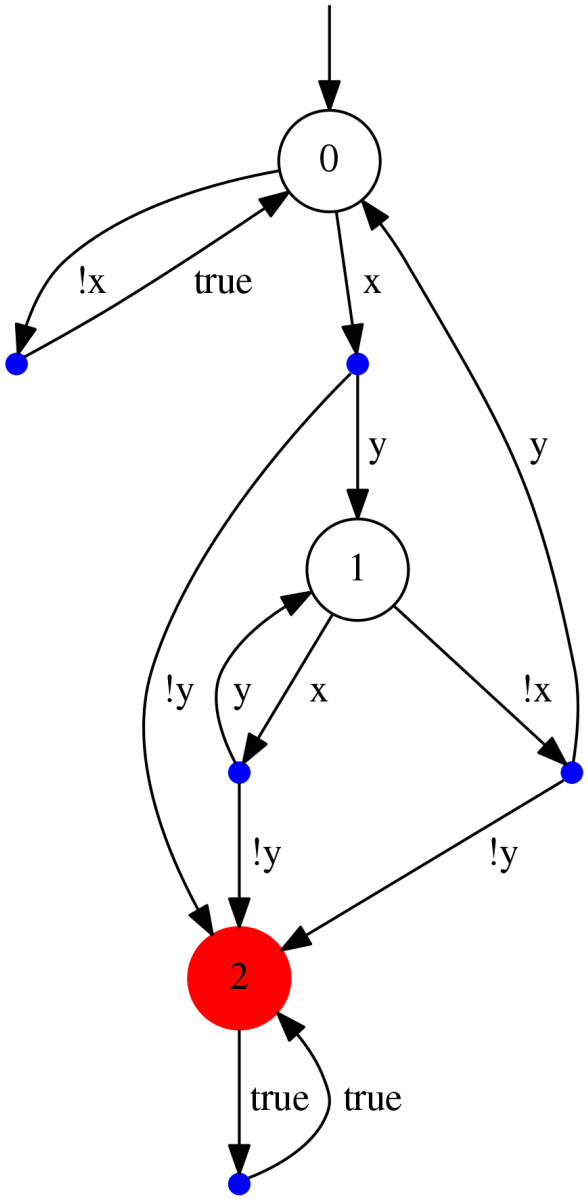

Blue states are the ones where the controller makes choice. A winning strategy avoids red states

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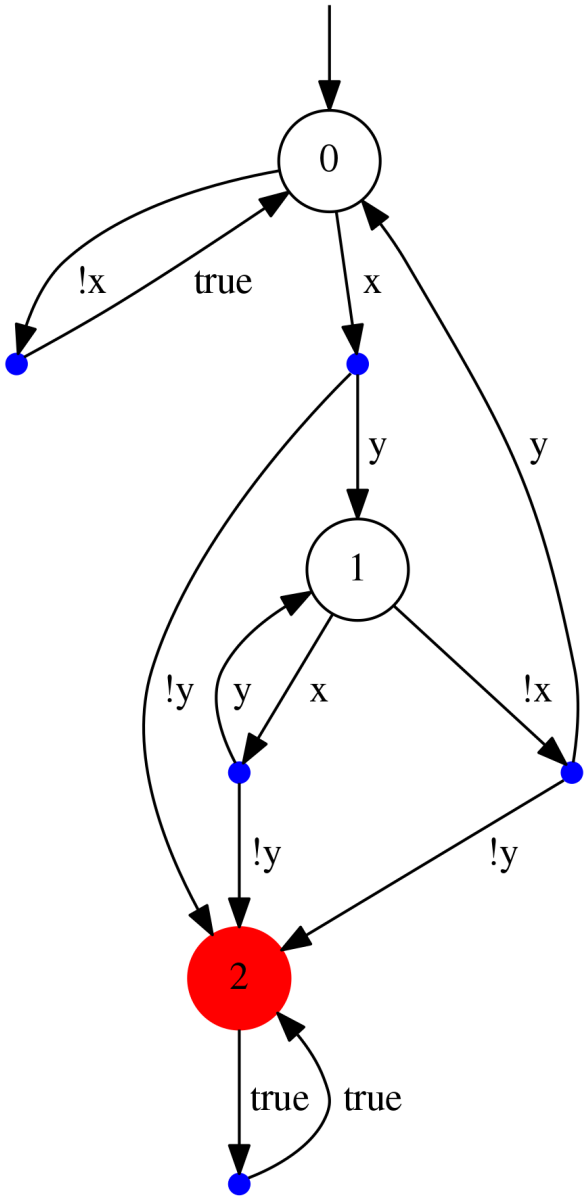

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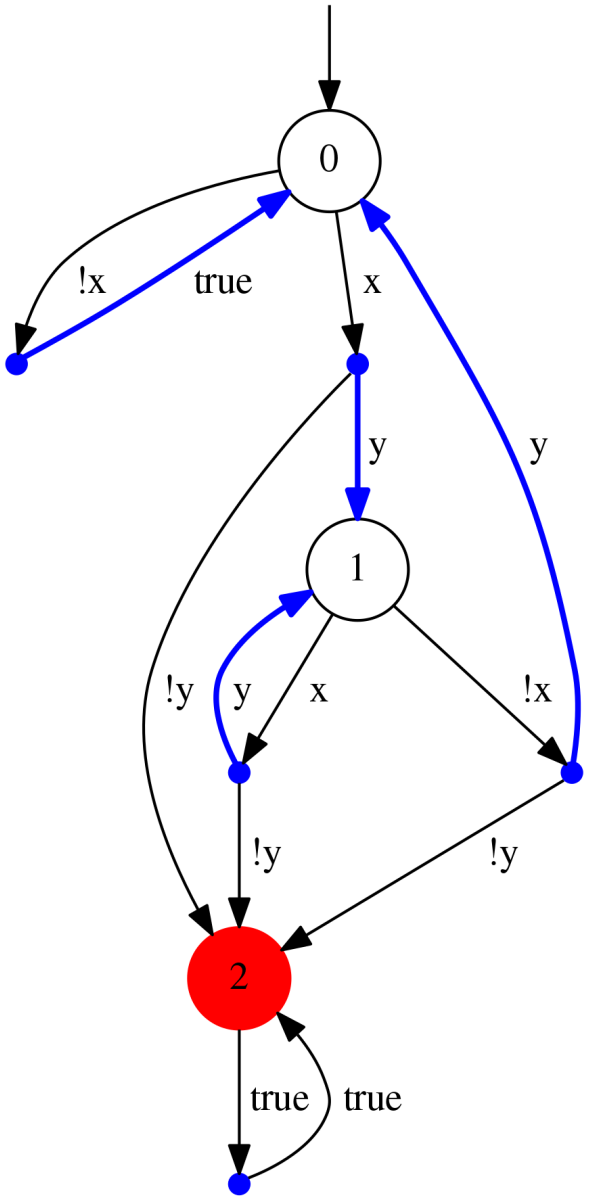

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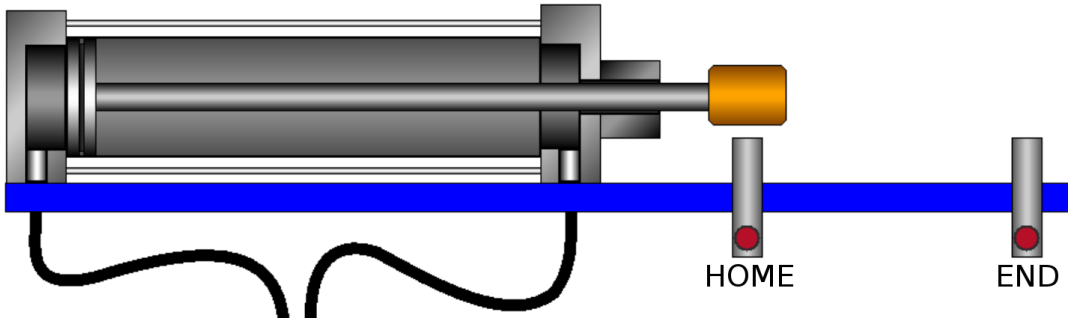
- A safety game is the same as invariance game for non-rejecting states

- There is a simple graph algorithm to find the winning strategy (i.e. the required controller)
- This algorithm can also find that there is no winning strategy, i.e. the environment can always force the state machine to enter the rejecting state

Cylinder example (1)



- Binary position (home, ¬home) and binary control signal (fwd, ¬fwd)
- Specification for the plant (the position on the next turn is determined by the control signal): $\mathbf{G}(\text{fwd} \leftrightarrow \mathbf{X}(\neg\text{home}))$
- We will require the controller to move the cylinder infinitely from one position to another: $\mathbf{G}(\text{home} \leftrightarrow \mathbf{X}(\neg\text{home}))$
- Let's put it together:

$$f = \mathbf{G}(\text{fwd} \leftrightarrow \mathbf{X}(\neg\text{home})) \rightarrow \mathbf{G}(\text{home} \leftrightarrow \mathbf{X}(\neg\text{home}))$$

- Is it a safety property?

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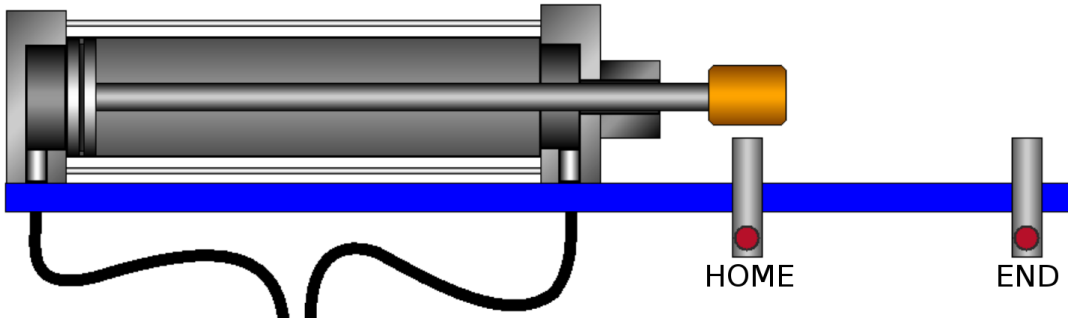

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- Is it a safety property? No! The environment can still violate plant assumptions even after the controller makes a mistake!

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- Direct approach
  - There is a more advanced method for non-safety formulae
  - If it is possible to convert the formula to a deterministic Büchi automaton, then the game-theoretical approach still applies with some modifications
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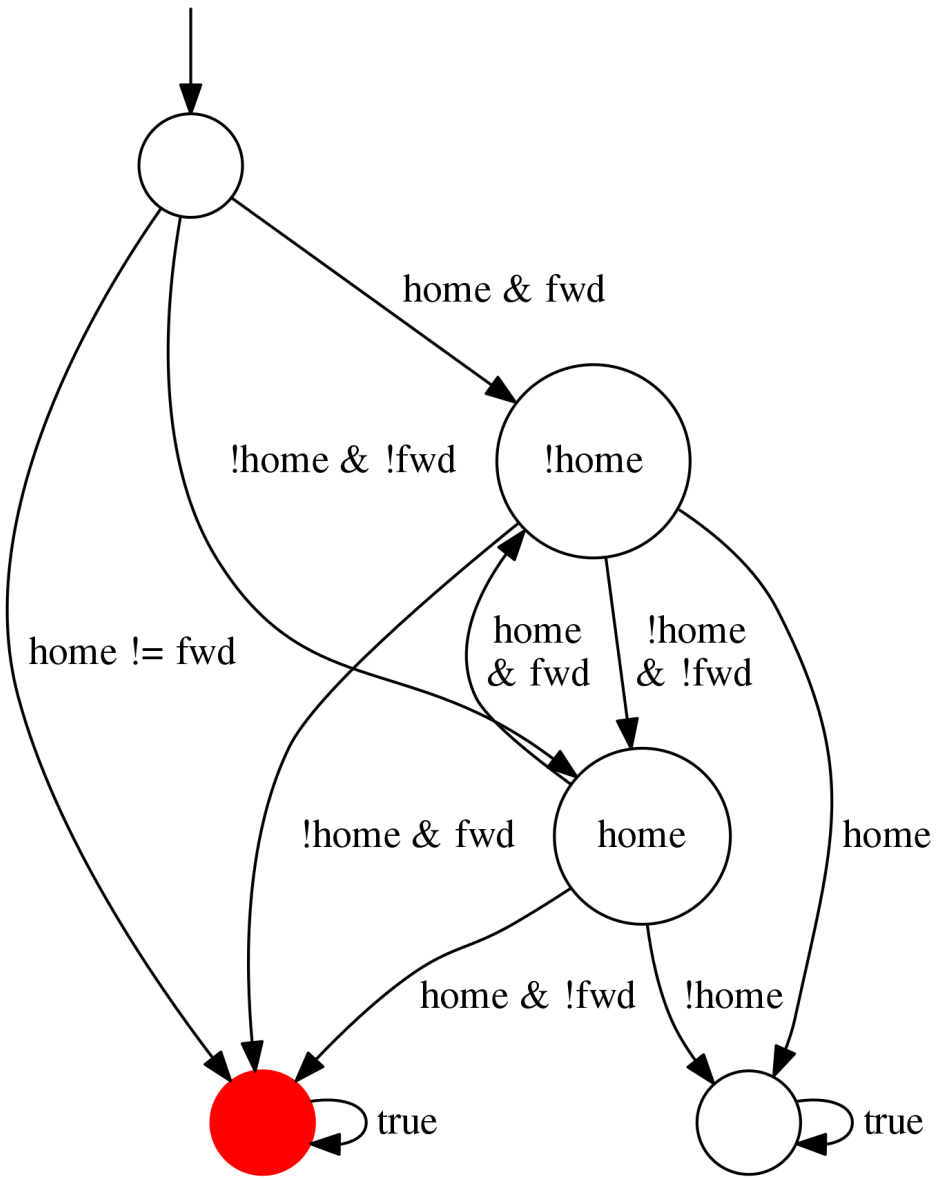
  - The controller should satisfy the requirement until the environment violates plant assumptions
  - $f' = (\text{home} \leftrightarrow \mathbf{X}(\neg\text{home}))\,\mathbf{W}\,\neg(\text{fwd} \leftrightarrow \mathbf{X}(\neg\text{home}))$
  - $\mathbf{W}$ – weak until: $x\,\mathbf{W}\,y = (x\,\mathbf{U}\,y) \vee (\mathbf{G}\,x)$

Safety automaton for the modified formula



Transform the automaton to a graph game and find the winning strategy for the controller

Literature

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3. S. Schewe, B. Finkbeiner. Bounded synthesis. In K.S. Namjoshi, T. Yoneda, T. Higashino, Y. Okamura (eds.) ATVA, LNCS, vol. 4762, pp. 474–488. Springer, 2007