Lecture 4. Formal specifications: LTL, CTL

ELEC-E8110 Automation Systems Synthesis and Analysis

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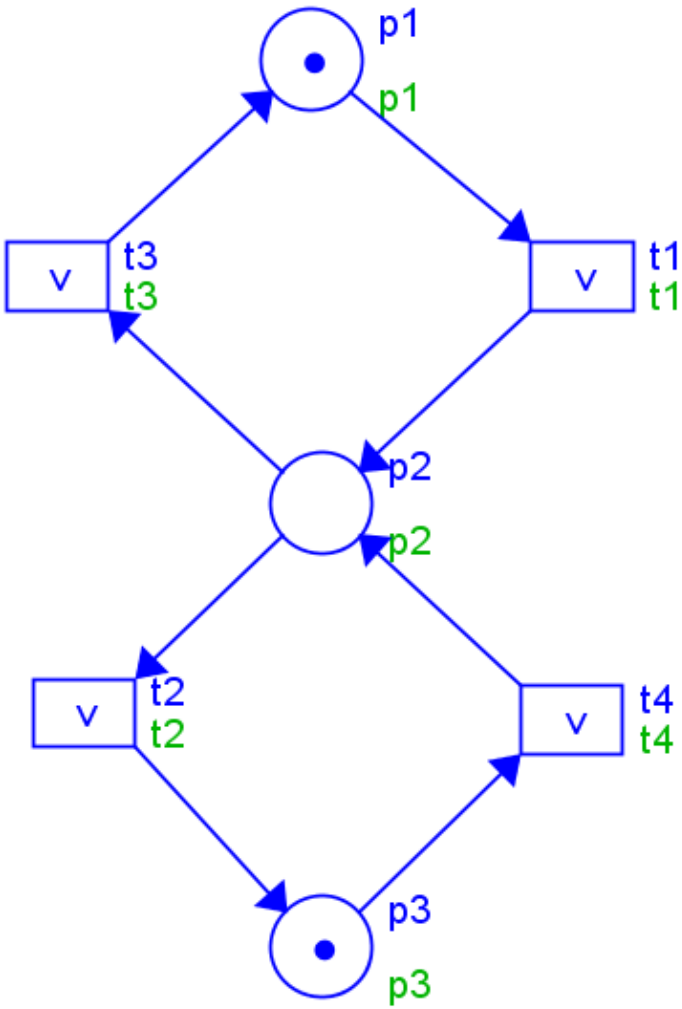
State (reachability) graph of a system

- Nodes: all reachable states of the system
- If the system is modular, then the state of the system consists of the state of all its modules
- Directed edges: one-step evolutions of the state
- Multiple outgoing edges are possible from each state, i.e. nondeterminism is common

State graph: example



NCES module (actually, a Petri net)

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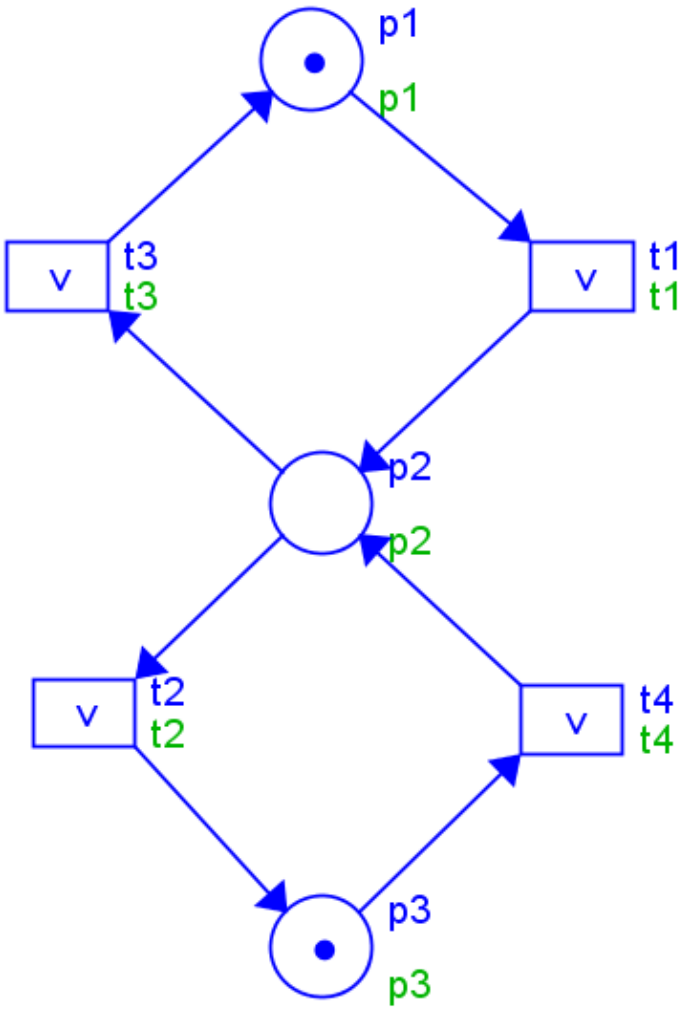

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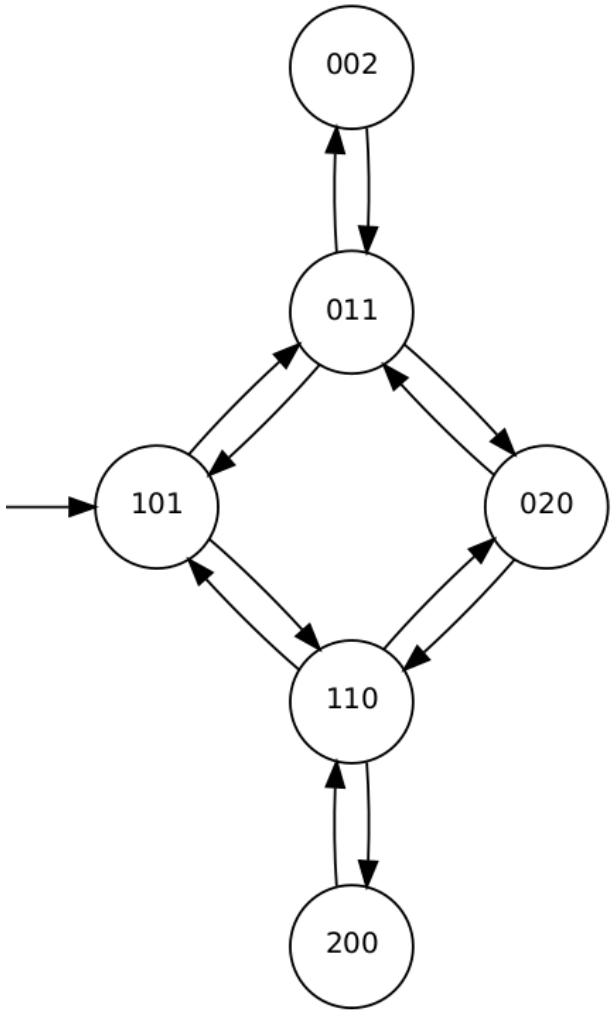
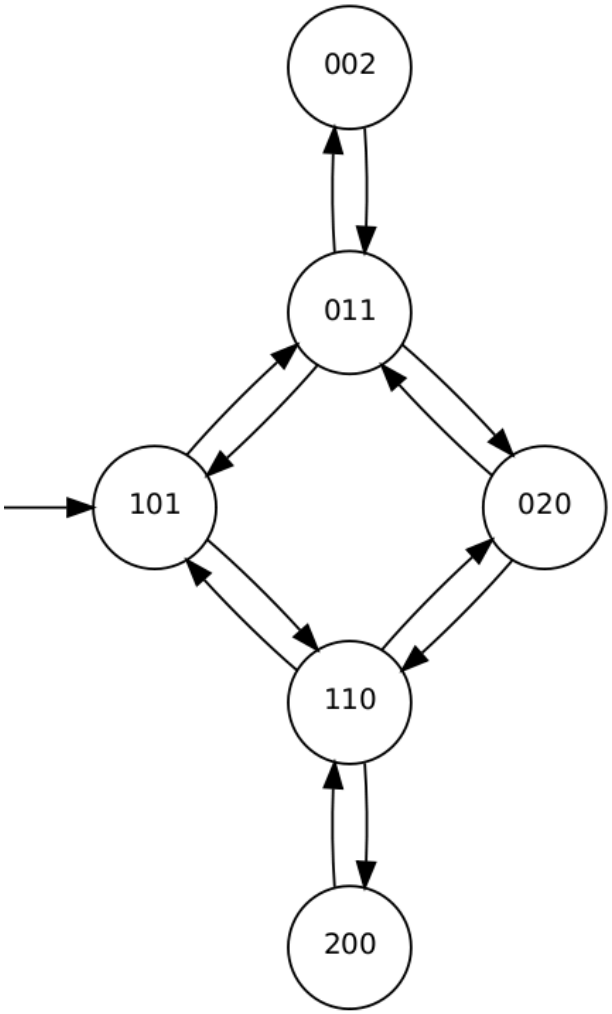


State graph, state $= p_1 p_2 p_3$

Kripke structures

- Formalization of a state graph
- Let AP be the a finite set of so-called **atomic propositions**
- Then $M = (S, I, T, L)$ is a **Kripke structure**, where:
- S is a finite set of **states**
- $I \subset S$ is a set of **initial states**
- $T \subset S \times S$ is a **transition relation**
- $L : S \to 2^{\mathrm{AP}}$ is a **labeling function**
- **No deadlock assumption:** $\forall s \in S \, \exists s' \in S : (s, s') \in T$

State graph interpreted as a Kripke structure



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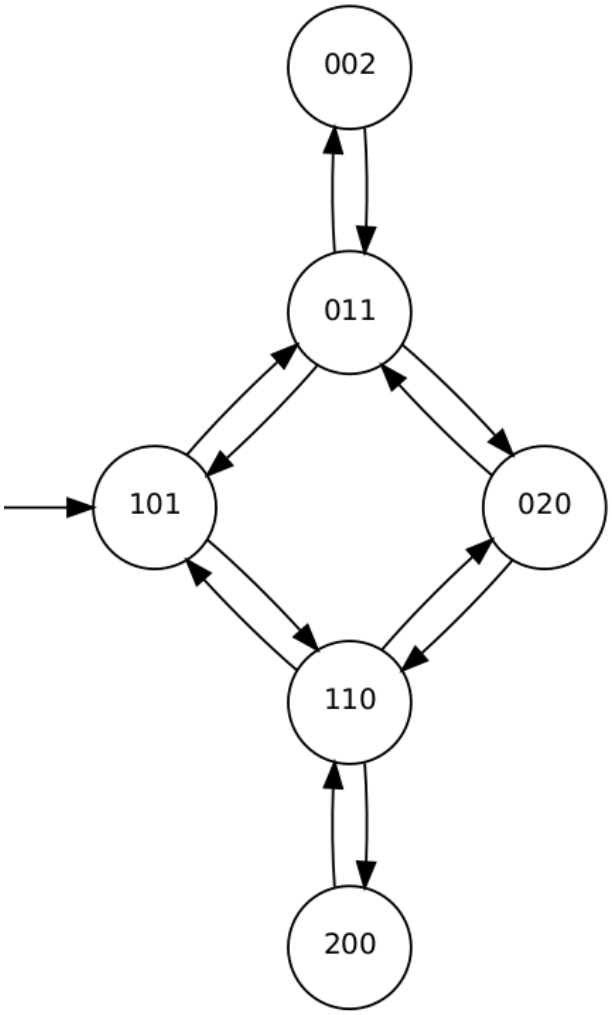

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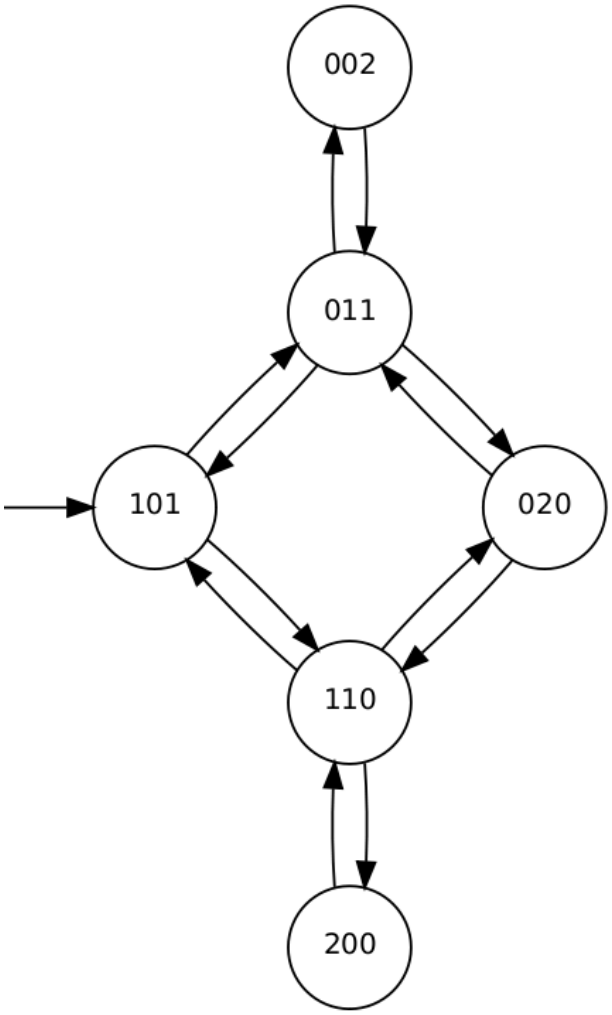

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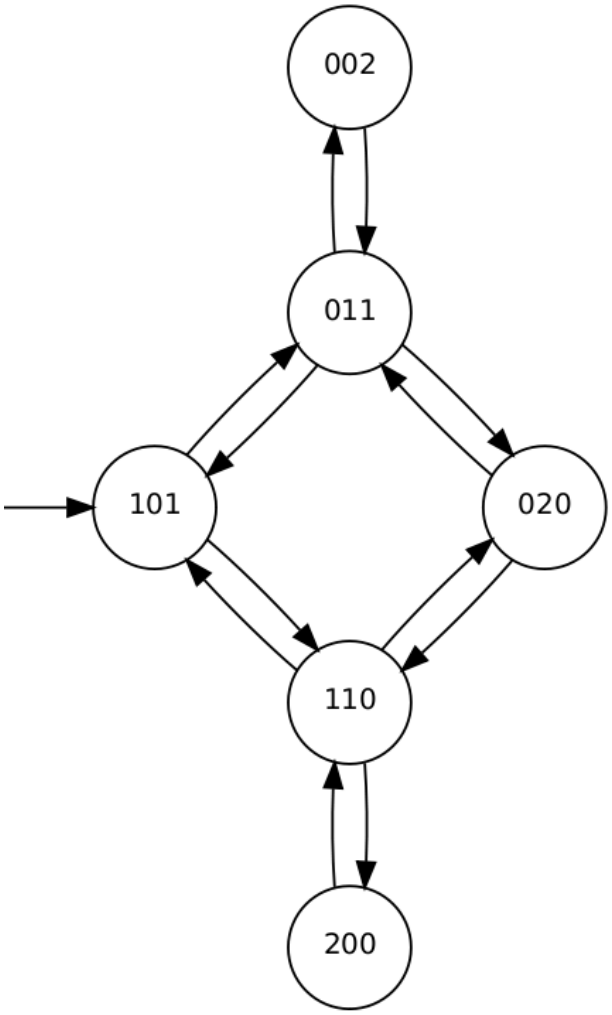
- S:

State graph interpreted as a Kripke structure



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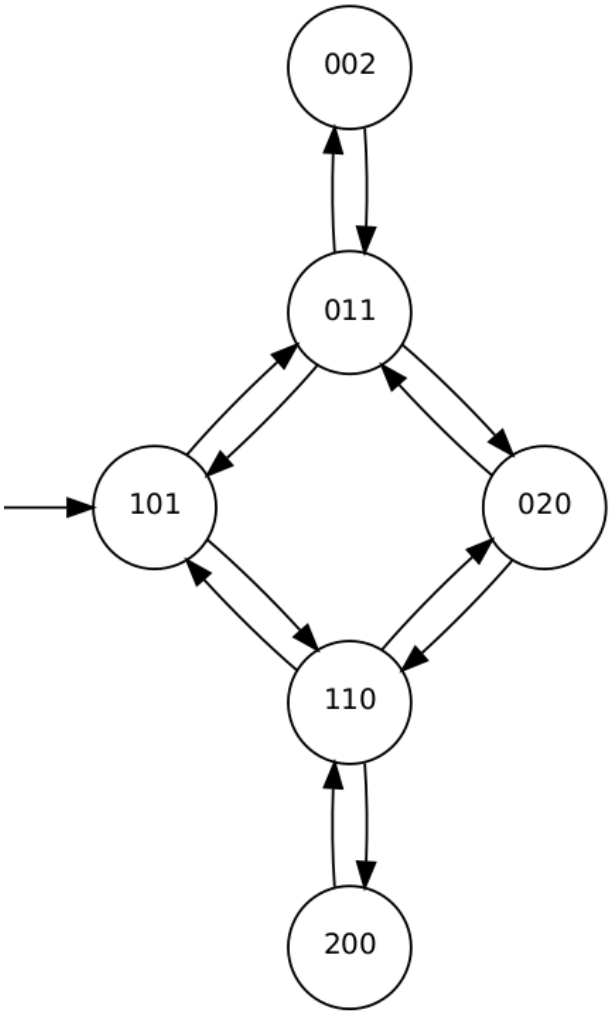

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- Note: in UPPAAL and NCES models there is always one initial state!
- $T \subset S \times S$:

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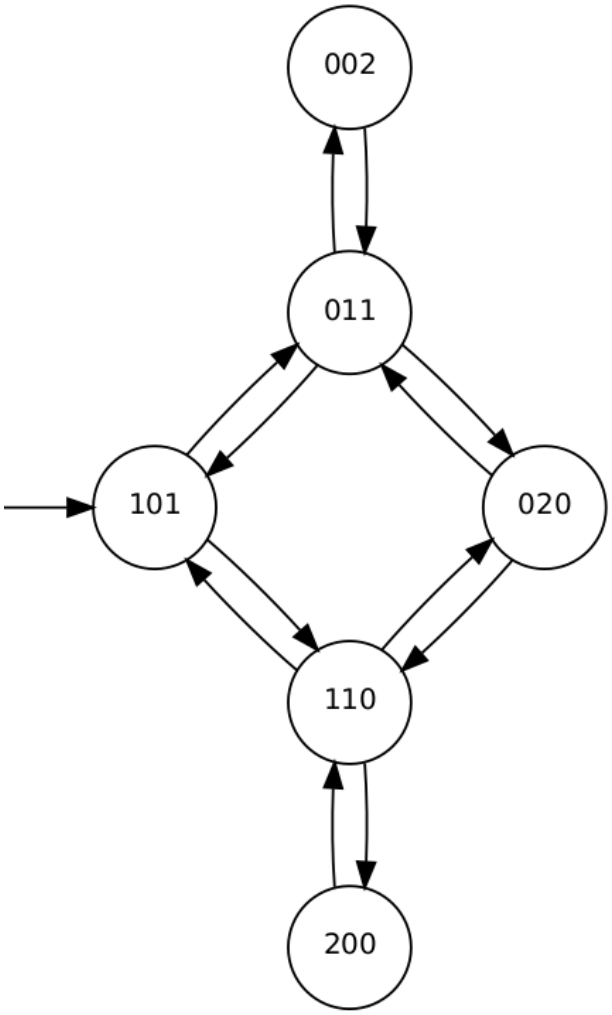

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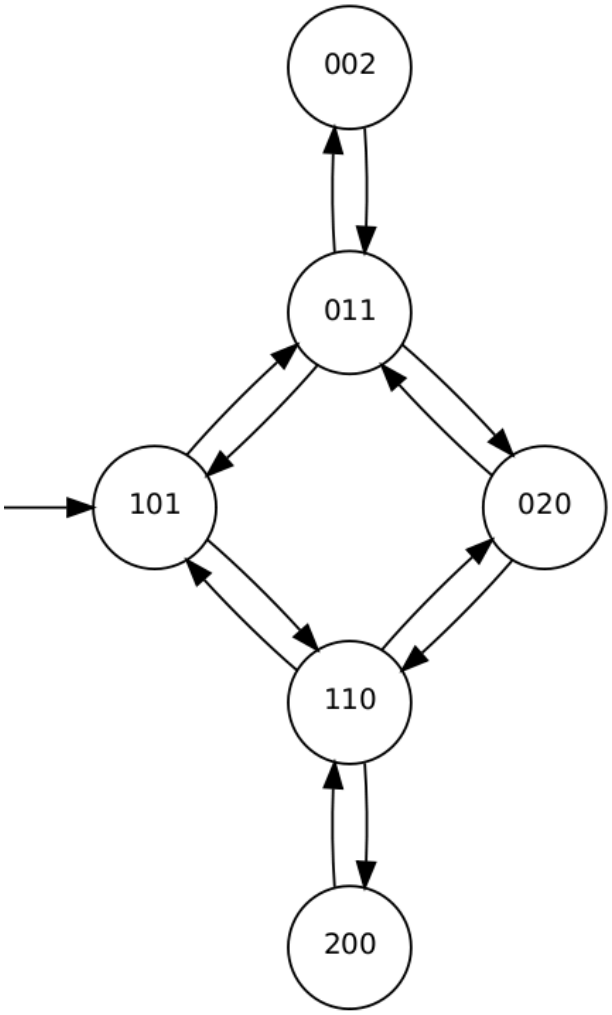

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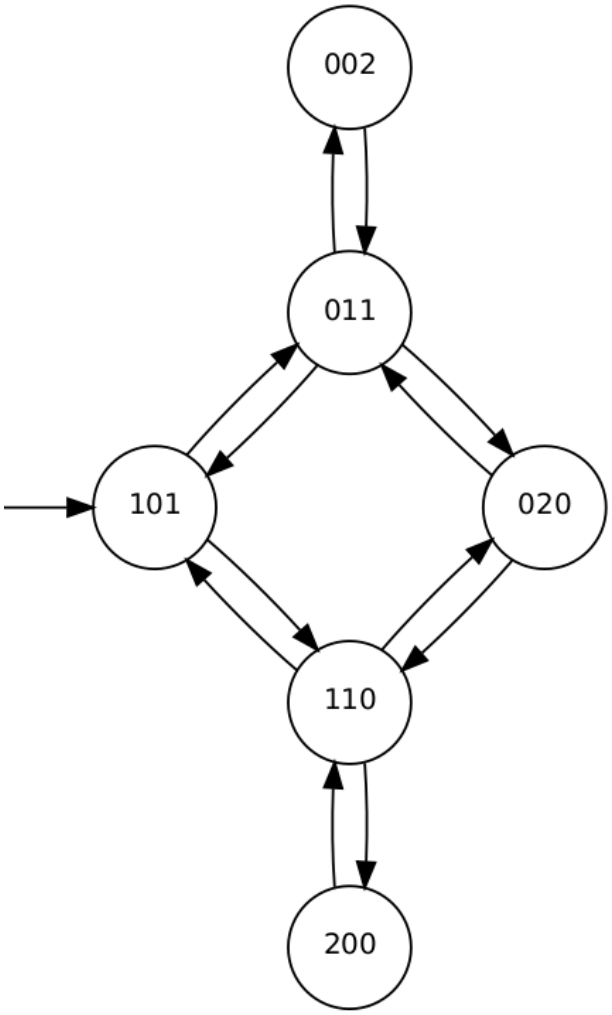

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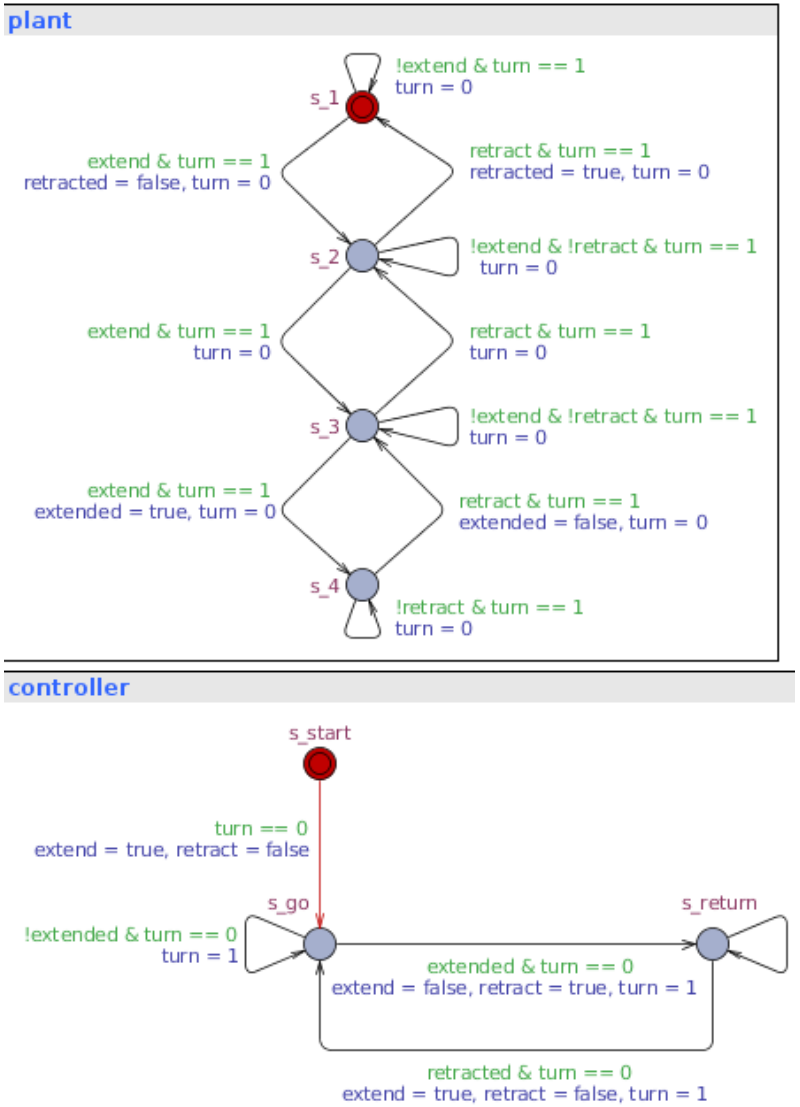
- **Specifications** can be interpreted as predicates over Kripke structures

Kripke structures / state graphs for UPPAAL models?



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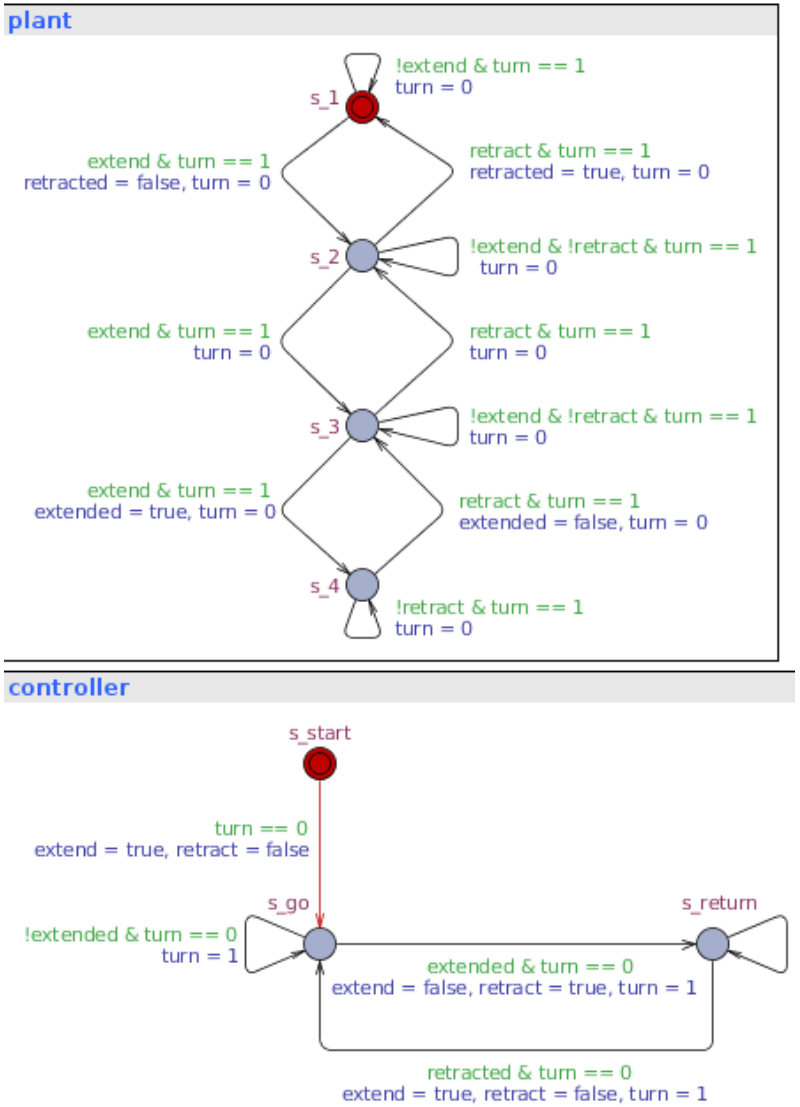

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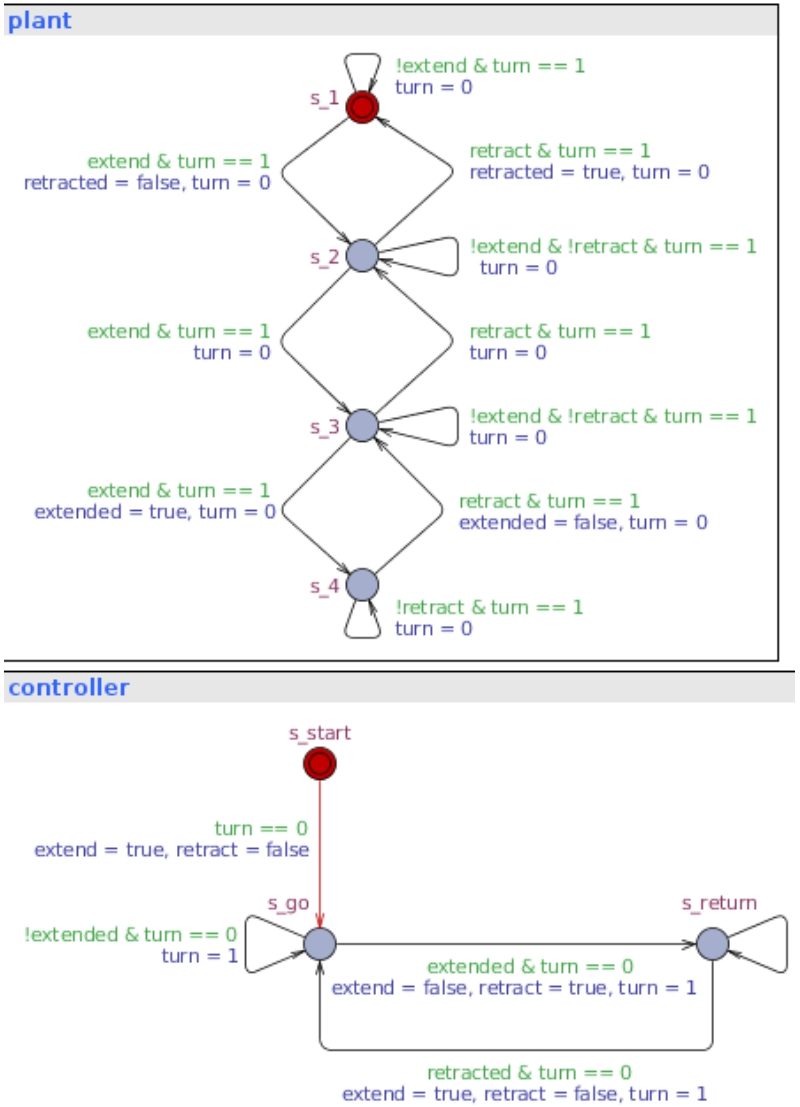

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- $I \subset S$:

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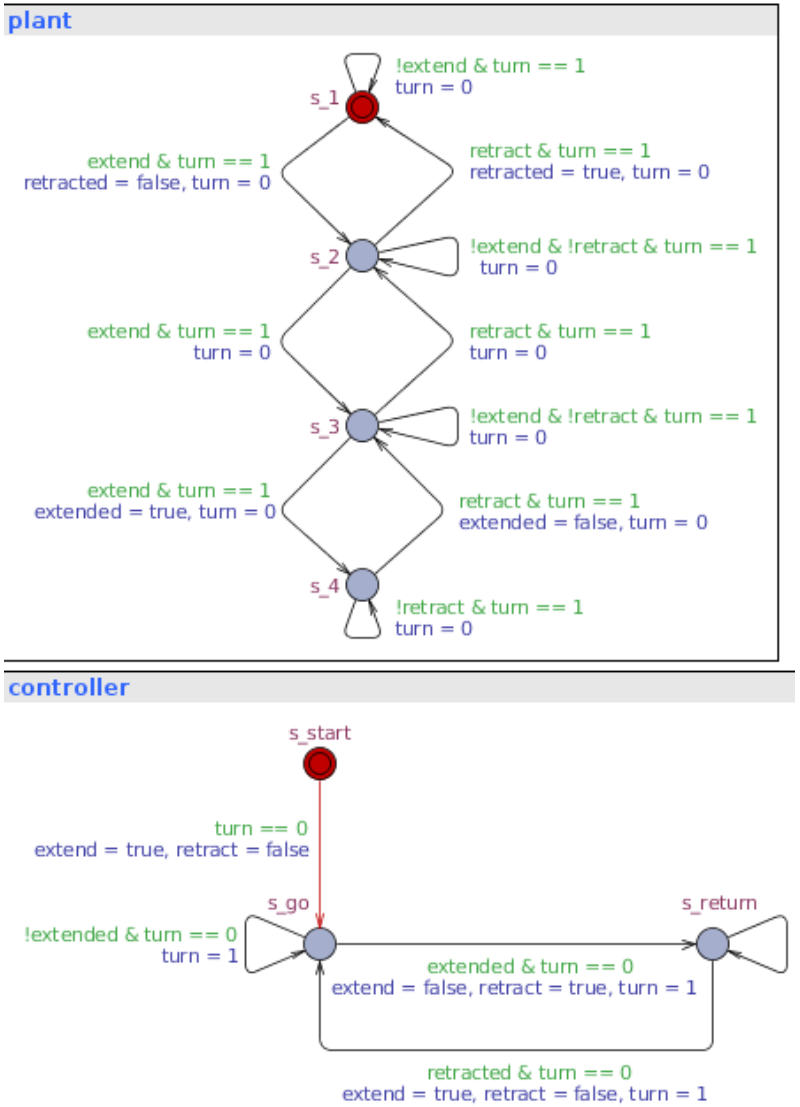

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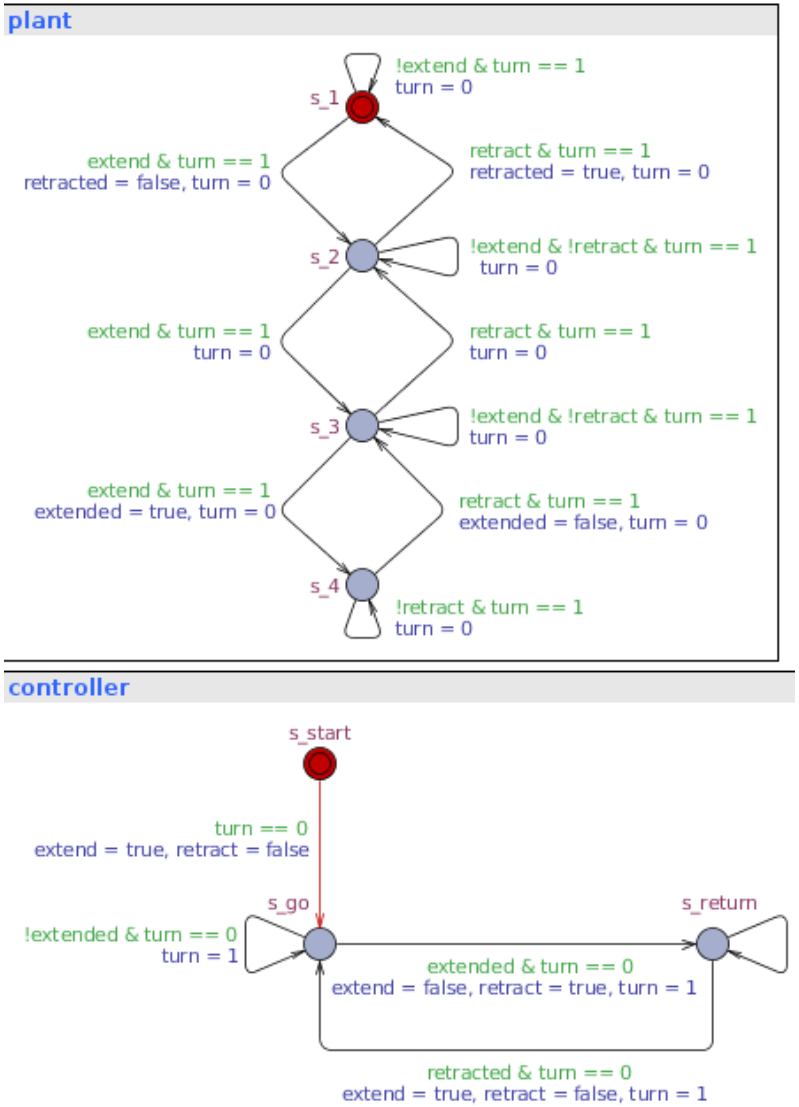

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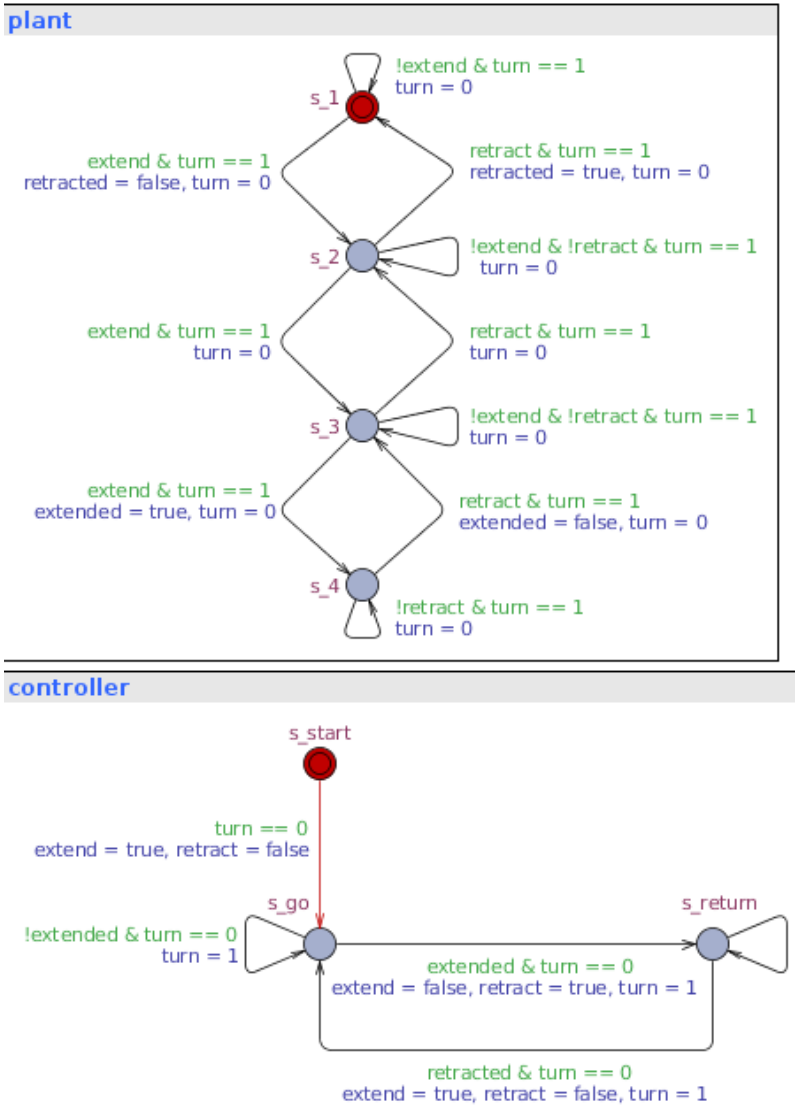

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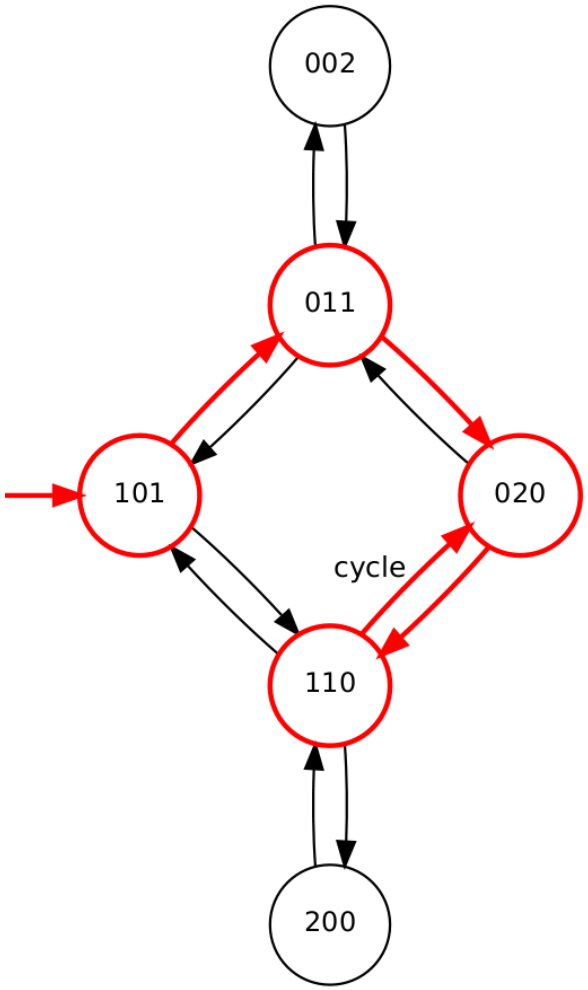
- Note: we ignore timed capabilities of UPPAAL by now

System behaviors are paths in Kripke structures



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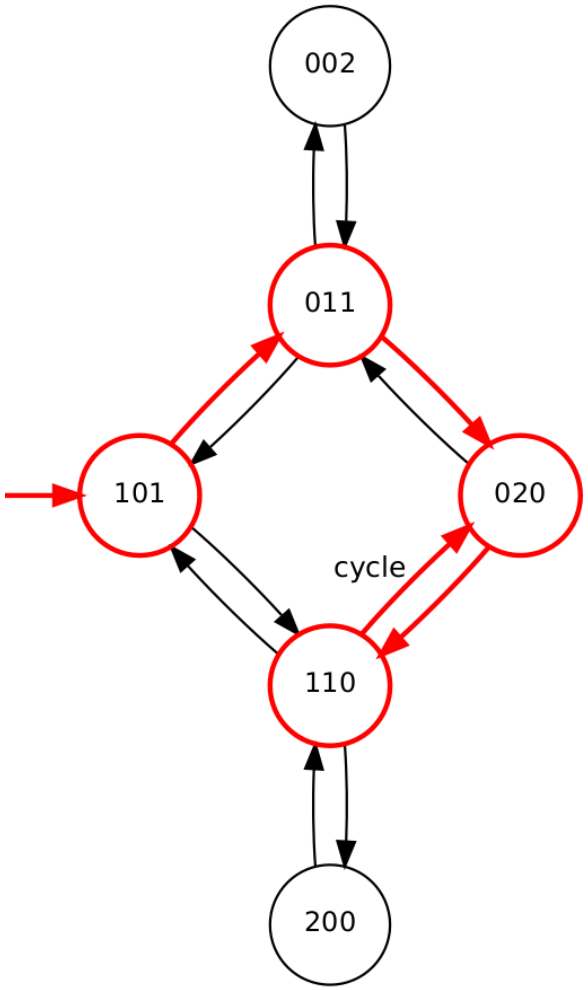

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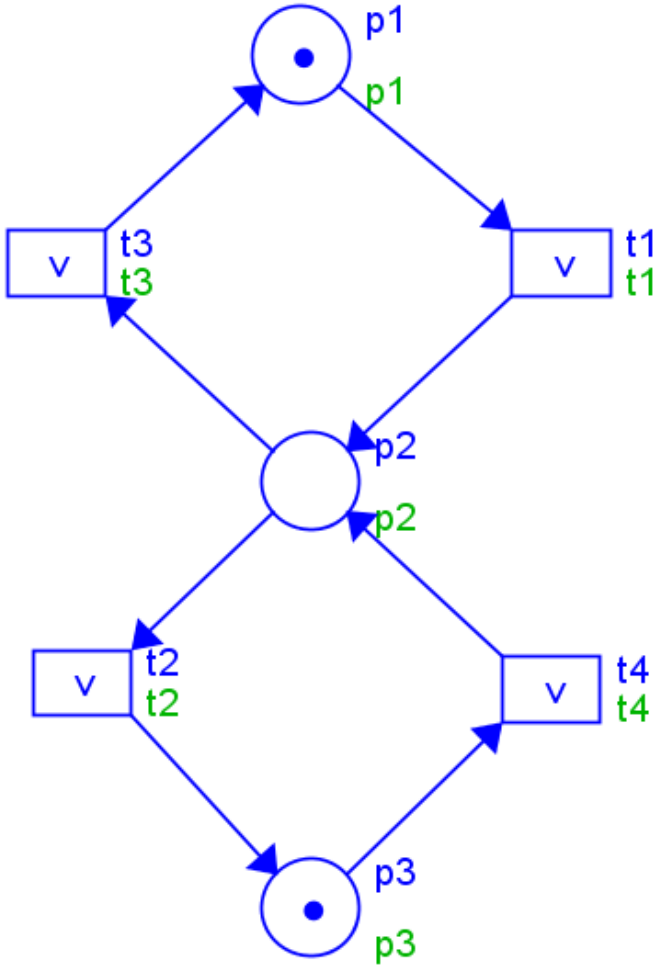

What happens in terms of the original system?

Single behavior view

- Assume that now we have only two atomic propositions: p and q
- All possible behaviors are infinite sequences over $2^{\{p,q\}}$
- Example: $\{p, q\}, \{p\}, \{\}, \text{cycle}(\{q\}, \{p, q\})$

Single behavior view

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- All possible behaviors are infinite sequences over $2^{\{p,q\}}$
- Example: $\{p, q\}, \{p\}, \{\}, \text{cycle}(\{q\}, \{p, q\})$
- Boolean logic is able to characterize single elements of such sequences
- Can we somehow introduce predicates over infinite sequences of atomic propositions?
- For example, to formulate a specification: each p is followed by $\neg p$ on the next step (which is false for the example)

Linear temporal logic (LTL)

- Formal language which extends the usual propositional Boolean logic
- Variables: atomic propositions, e.g. p and q
- Usual Boolean operators are allowed, e.g. $p \to q$ (i.e. $\neg p \vee q$) is an LTL formula, but it refers to the **first element** of an infinite sequence

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- **U**: **u**ntil (binary operator), e.g. $p\,\mathbf{U}\,q$ means "q must happen at some step, and the sequence must satisfy p until (non-inclusive) q happens"

Examples of LTL formulae

- Path 1: $\{p, q\}, \{p\}, \{\}, \text{cycle}(\{q\}, \{p, q\})$
- Path 2: $\text{cycle}(\{p, q\})$
- Path 3: $\{\}, \text{cycle}(\{p\}, \{p, q\}, \{q\})$

- $f_1 = \mathbf{G}\, p$

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- $f_4 = \mathbf{X}\,\mathbf{X}\,\mathbf{X}\,\mathbf{X}\, p$ ("on the fifth step") – paths 1, 2, 3
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- $f_7 = \mathbf{G}(p \rightarrow \mathbf{X}\, q)$

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- $f_7 = \mathbf{G}(p \rightarrow \mathbf{X}\, q)$ ("p is always followed by q") – paths 2, 3

Path visualization tool

- Download from `https://github.com/igor-buzhinsky/nusmv_counterexample_visualizer`
- The tool supports interpreting counterexamples produced by NuSMV (will be covered later in the course), but can also be used to **structurally explain** LTL formula values on user-specified paths
- **Important atomic propositions** are highlighted

- Variable view: suppose that the state of the formal model is composed on a number of variables, either Boolean or integer
- Boolean variables can be interpreted as atomic propositions right away
- Statements over integer variables (e.g. comparisons like $x > 5$) can also be interepreted as atomic propositions

Path visualization tool: example

- Visualize $\mathbf{G}(p \to \mathbf{X}\, q)$ on path $\{p, q\}, \{p\}, \{\}, \text{cycle}(\{q\}, \{p, q\})$:

#	Value	LTL specification
7	TRUE	G F (p & q)
8	TRUE	G F (p & q)
9	TRUE	G F (p & q)
10	FALSE	G (p -> X q)
11	TRUE	G (p -> X q)
12	TRUE	G (p -> X q)

```
step 0  G((p -> X(q)))
prefix  ‖ ‖T     ‖ T‖‖‖
        ‖ ‖      ‖FF‖‖‖
        ‖ ‖FFFFFFFFF‖‖
        ‖FFFFFFFFFFF‖
step 1  G((p -> X(q)))
prefix  ‖ ‖T     ‖ F‖‖‖
        ‖ ‖      ‖FF‖‖‖
        ‖ ‖FFFFFFFFF‖‖
        ‖FFFFFFFFFFF‖
step 2  G((p -> X(q)))
prefix  ‖ ‖F     ‖ F‖‖‖
        ‖ ‖      ‖TT‖‖‖
```

Step	p	q
0 prefix	TRUE	TRUE
1 prefix	TRUE	FALSE
2 prefix	FALSE	FALSE
3 loop	FALSE	TRUE
4 loop	TRUE	TRUE

Path visualization tool: input format

```
-- specification  G (p -> X q)
  -> new state <-
    p = TRUE
    q = TRUE
  -> new state <-
    p = TRUE
    q = FALSE
  -> new state <-
    p = FALSE
    q = FALSE
  -- Loop starts here
  -> new state <-
    p = FALSE
    q = TRUE
  -> new state <-
    p = TRUE
    q = TRUE
```

LTL: simplification and equivalence rules

- $\mathbf{G}\,\mathbf{G}\,f = \mathbf{G}\,f$
- $\mathbf{F}\,\mathbf{F}\,f = \mathbf{F}\,f$

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LTL: simplification and equivalence rules

- $\mathbf{G}\,\mathbf{G}\,f = \mathbf{G}\,f$
- $\mathbf{F}\,\mathbf{F}\,f = \mathbf{F}\,f$

- $\mathbf{G}\,\mathbf{X}\,f = \mathbf{X}\,\mathbf{G}\,f$
- $\mathbf{F}\,\mathbf{X}\,f = \mathbf{X}\,\mathbf{F}\,f$

- $\neg\,\mathbf{G}(f) = \mathbf{F}(\neg f)$
- $\neg\,\mathbf{F}(f) = \mathbf{G}(\neg f)$

LTL verification: definition

- Kripke structure M satisfies LTL formula f (written: $M \models f$), if **all paths** in M which **start in** M**'s initial states** satisfy f

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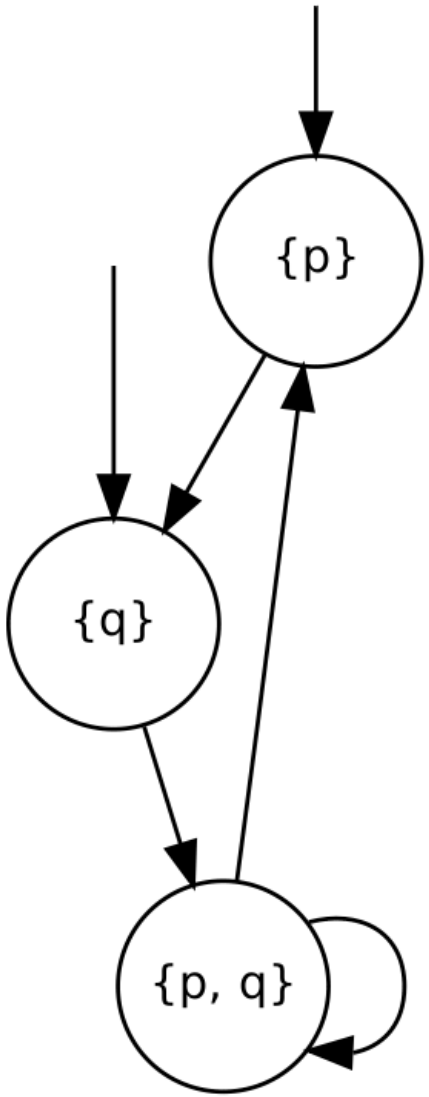
Quiz: which of these LTL formulae are satisfied by the KS on the right? Why?

- $f_1 = \mathbf{G}\, p$
- $f_2 = \mathbf{F}(\neg p \wedge \neg q)$
- $f_3 = p\, \mathbf{U}(\neg p \wedge \neg q)$
- $f_4 = \mathbf{X}\,\mathbf{X}\,\mathbf{X}\,\mathbf{X}\, p$
- $f_5 = \mathbf{F}\,\mathbf{G}(p \wedge q)$
- $f_6 = \mathbf{G}\,\mathbf{F}(p \wedge q)$
- $f_7 = \mathbf{G}(p \rightarrow \mathbf{X}\, q)$



Quiz answers

Quiz answers

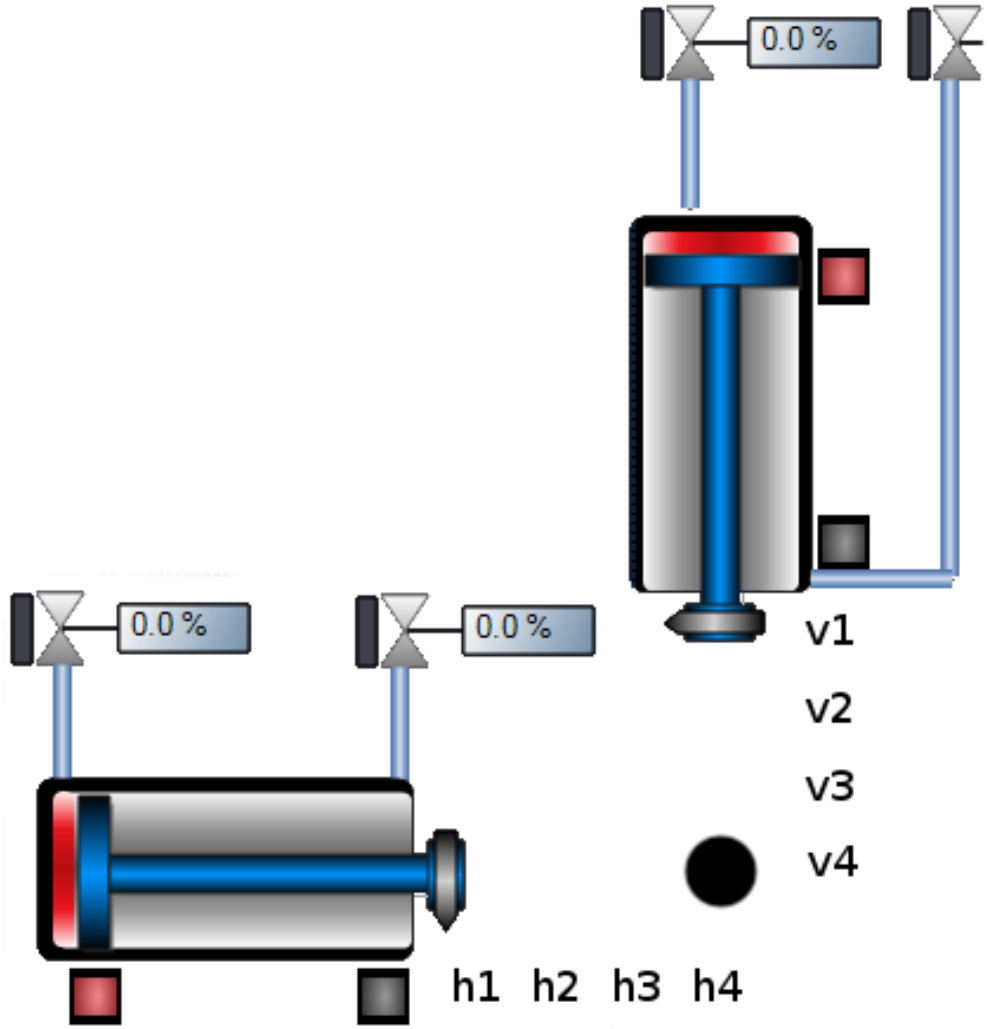
- Only $f_6 = \mathbf{G}\,\mathbf{F}(p \wedge q)$

LTL verification: **model checking** algorithm

- We wish to check whether f holds for Kripke structure M
- Automata-theoretic approach
- $\neg f$ is converted to a so-called **Büchi automaton**, which is an acceptor over infinite words which satisfy $\neg f$
- M is composed with this automaton
- If the composition accepts at least one infinite word, then this word satisfies $\neg f$ and belongs to M, so f is false, and the obtained word is a **counterexample**
- Otherwise, f is true

- We won't go into details

Two cylinders system



- The extension of each cylinder is discretized into four intervals
- When both cylinders share interval 4, they collide
- A workpiece can be placed into the shared interval
- If a cylinder reaches interval 4 and there is a workpiece, it is pushed

LTL specification for the two cylinders system: plant model

- Atomic propositions: h_1, h_2, h_3, h_4 (displacements of the horizonal cylinder), v_1, v_2, v_3, v_4 (displacements of the vertical cylinder), w (workpiece is present)
- Cylinder has a position:

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  $\mathbf{G}(\neg(v_1 \wedge v_2) \wedge \neg(v_1 \wedge v_3) \wedge \neg(v_1 \wedge v_4) \wedge \neg(v_2 \wedge v_3) \wedge \neg(v_2 \wedge v_4) \wedge \neg(v_3 \wedge v_4))$
- If a cylinder is fully extended, then there is no workpiece:

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  $\mathbf{G}(\neg(v_1 \wedge v_2) \wedge \neg(v_1 \wedge v_3) \wedge \neg(v_1 \wedge v_4) \wedge \neg(v_2 \wedge v_3) \wedge \neg(v_2 \wedge v_4) \wedge \neg(v_3 \wedge v_4))$
- If a cylinder is fully extended, then there is no workpiece:
  $\mathbf{G}(h_4 \vee v_4 \rightarrow \neg w)$
- ...

- Such specifications can help "debug" the plant model

LTL specification for the two cylinders system: requirements for the controller

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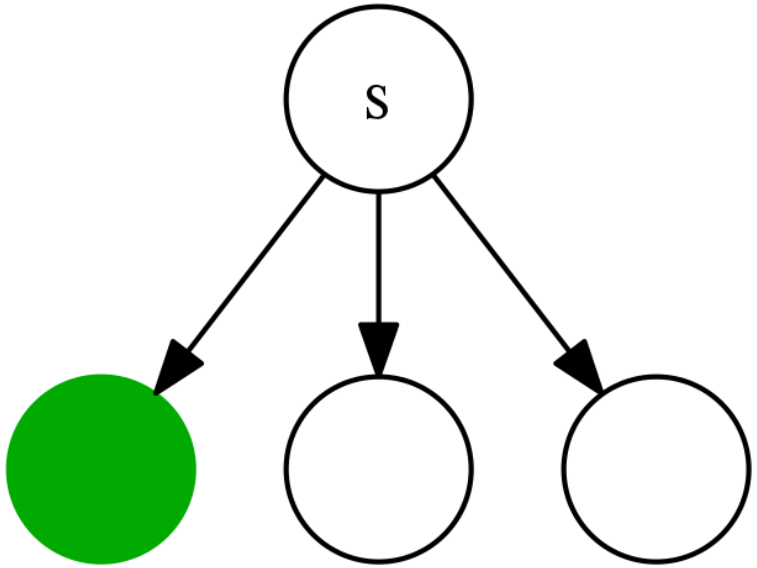
- When a workpiece appears, it must be eventually pushed away: $\mathbf{G}(w \rightarrow \mathbf{F}\,\neg w)$
- Cylinders iterate (each new workpiece is pushed by a different cylinder): $\mathbf{G}((h_4 \wedge (\mathbf{X}\,\neg h_4) \wedge \mathbf{F}\,w) \rightarrow \mathbf{X}((\neg w \wedge \neg v_4 \wedge \neg h_4)\,\mathbf{U}(w \wedge (w\,\mathbf{U}(v_4 \wedge \neg h_4)))))$, and the same for the other cylinder

Computation tree logic (CTL)

- In LTL, there is always an implicit quantification over all paths starting in initial states
- In **CTL**, all temporal operators are annotated with quantifiers

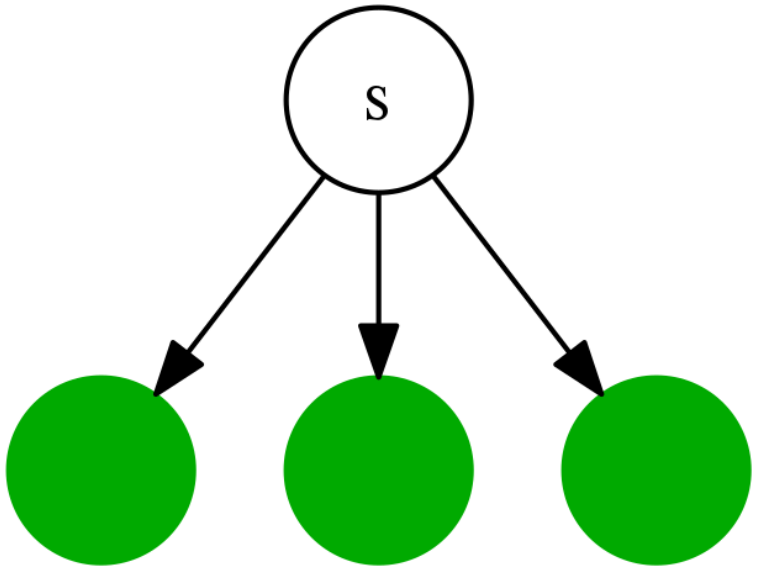
- CTL formulae characterize not infinite sequences, but rather states of the Kripke structure
- A Kripke structure satisfies a CTL formula, if **all its initial states** satisfy this formula
- Let s be a state of the KS, then $s \models f$ means s satisfies f

CTL: temporal operator **EX**



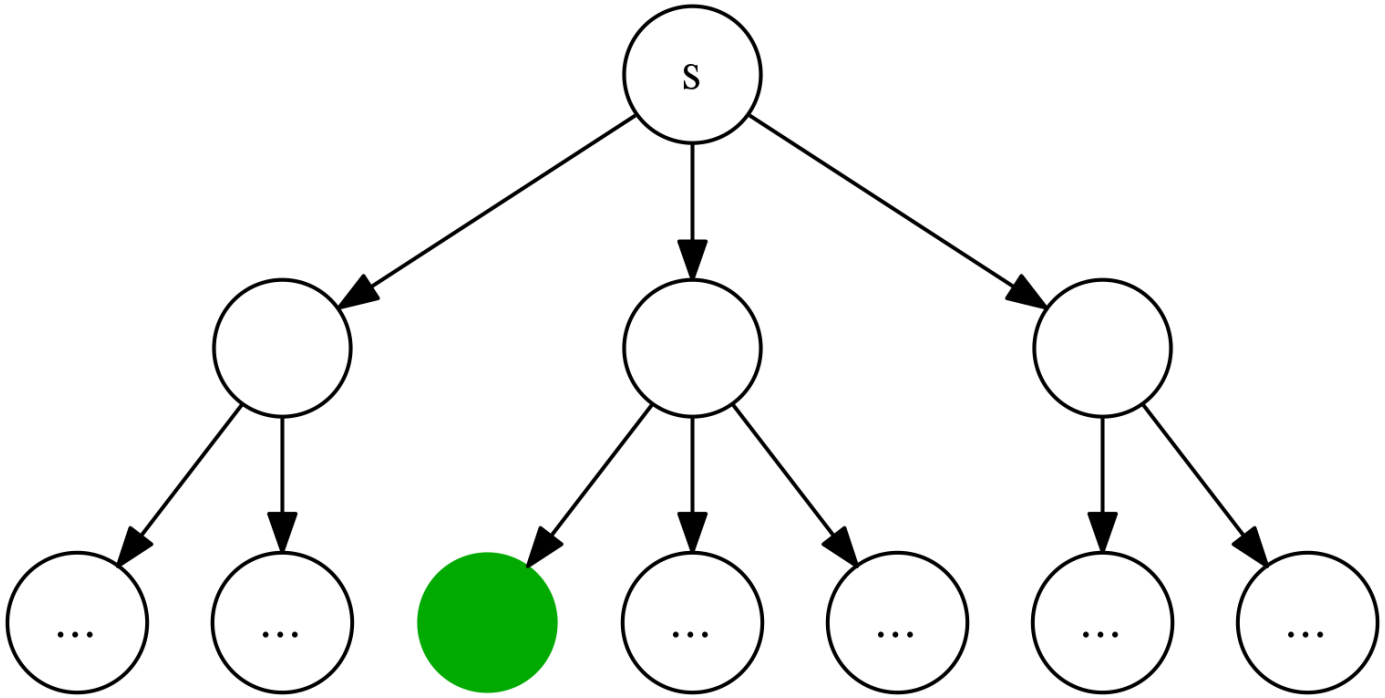
- $s \models \mathbf{EX}(f)$ ("exists next", not supported by UPPAAL): there is a successor of s where f holds

CTL: temporal operator **AX**



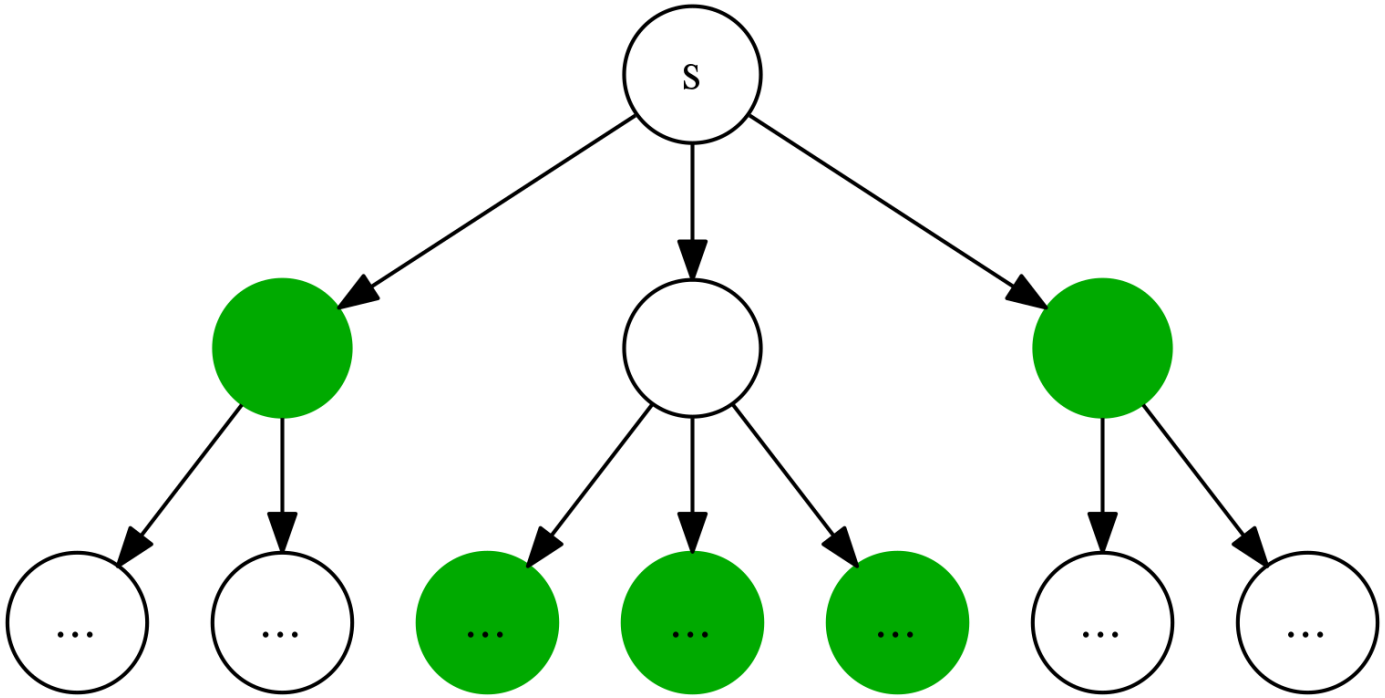
- $s \models \mathbf{AX}(f)$ ("for all next", not supported by UPPAAL): in all successors of s, f holds

CTL: temporal operator **EF**



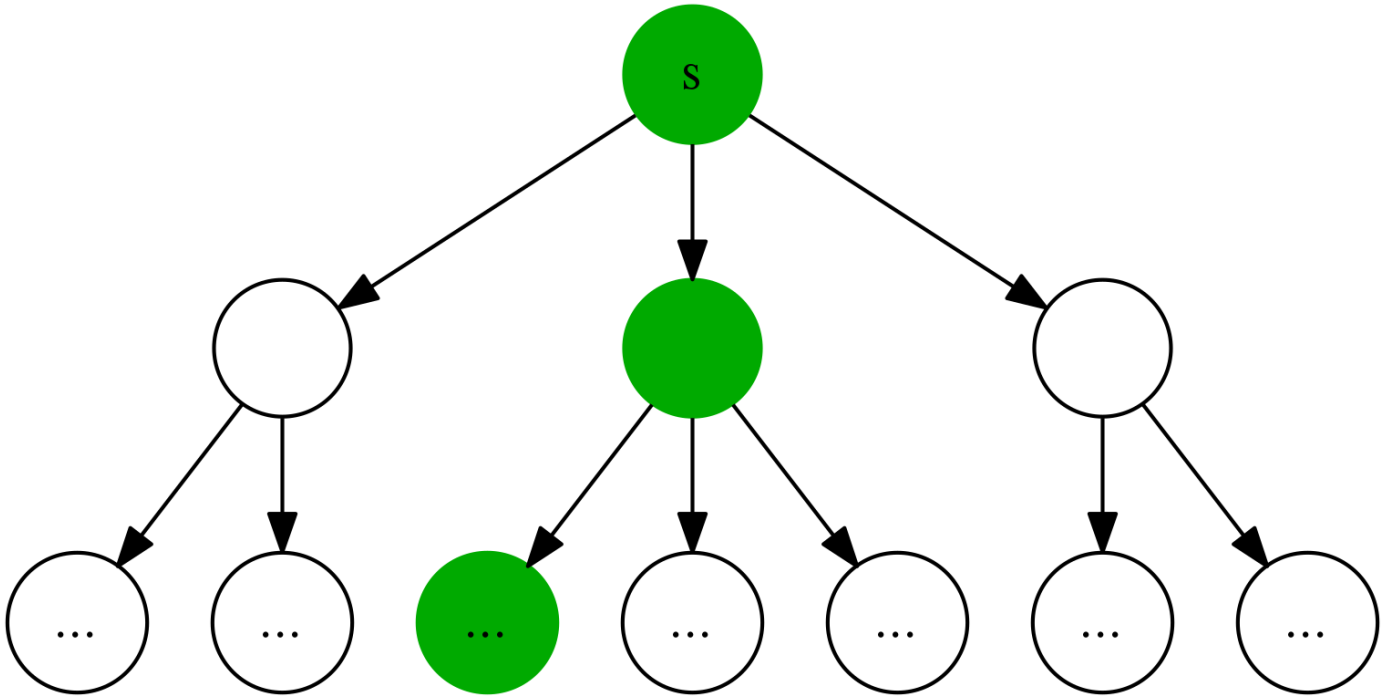
- $s \models \mathbf{EF}(f)$ ("exists in the future", E<> in UPPAAL): there exists a path starting in s such that f becomes valid at some point of this path

CTL: temporal operator **AF**



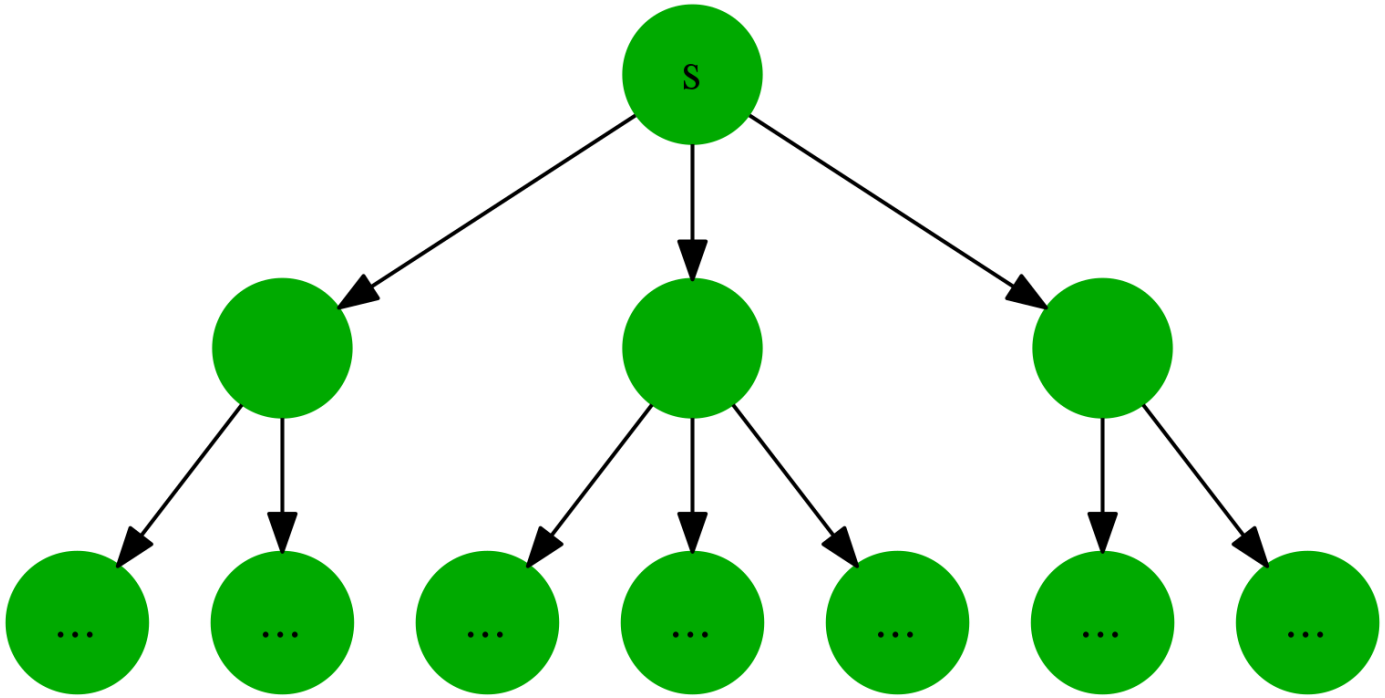
- $s \models \mathbf{AF}(f)$ ("for all in the future", A<> in UPPAAL): for all possible paths starting in s, f becomes true at some point

CTL: temporal operator **EG**



- $s \models \mathbf{EG}(f)$ ("exists globally", E[] in UPPAAL): there exists a path starting in s such that f holds at every state along this path

CTL: temporal operator **AG**



- $s \models \mathbf{AG}(f)$ ("for all globally", A[] in UPPAAL): for all possible paths starting in s, f is always true

CTL: temporal operators **EU** and **AU**

- $s \models f$ **EU** g ("exists until", not supported by UPPAAL): there exists a path starting in s such that f holds until (non-inclusive) g, and g eventually happens
- $s \models f$ **AU** g ("for all until", not supported by UPPAAL): for all possible paths starting in s, f holds until (non-inclusive) g, and g eventually happens

Are these formulae syntactically correct in LTL or CTL?

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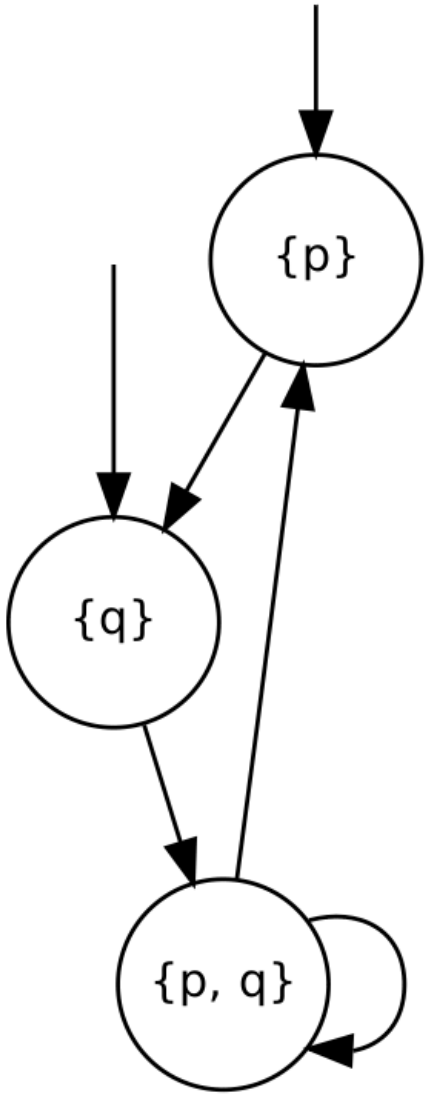
- $f_3 = \mathbf{AF}(p \wedge q)$
- $f_4 = \mathbf{EF}(\neg p \wedge \neg q)$
- $f_5 = \mathbf{AX}\,\mathbf{AX}\,\mathbf{AX}\,\mathbf{AX}\, p$
- $f_6 = \mathbf{EF}\,\mathbf{EG}(p \wedge q)$
- $f_7 = \mathbf{EG}\,\mathbf{EF}(p \wedge q)$
- $f_8 = \mathbf{AG}(p \rightarrow \mathbf{AX}\, q)$



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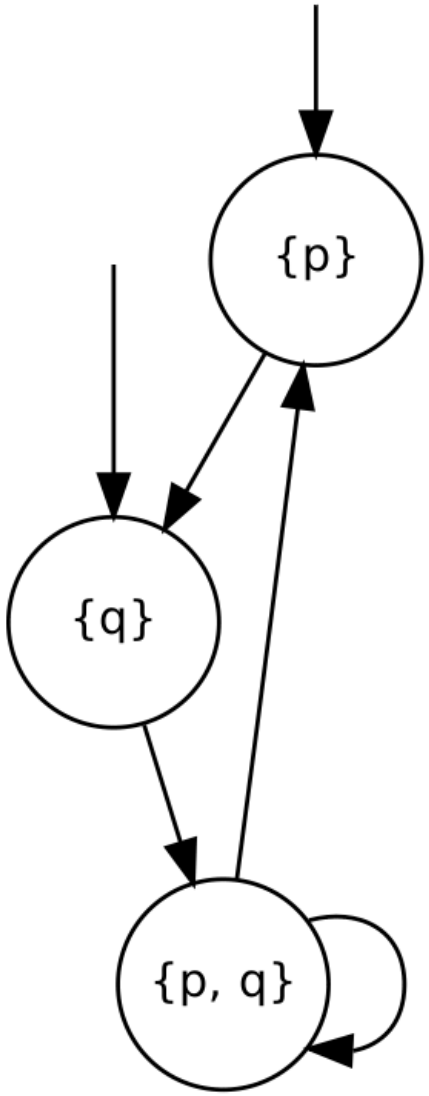
- Answer:



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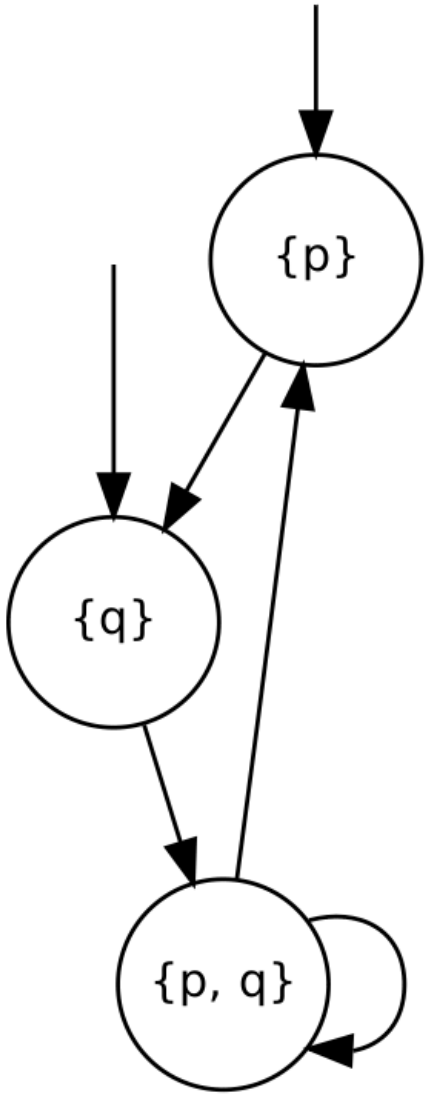
- Answer: f_2, f_3, f_6, f_7



CTL verification: model checking algorithms

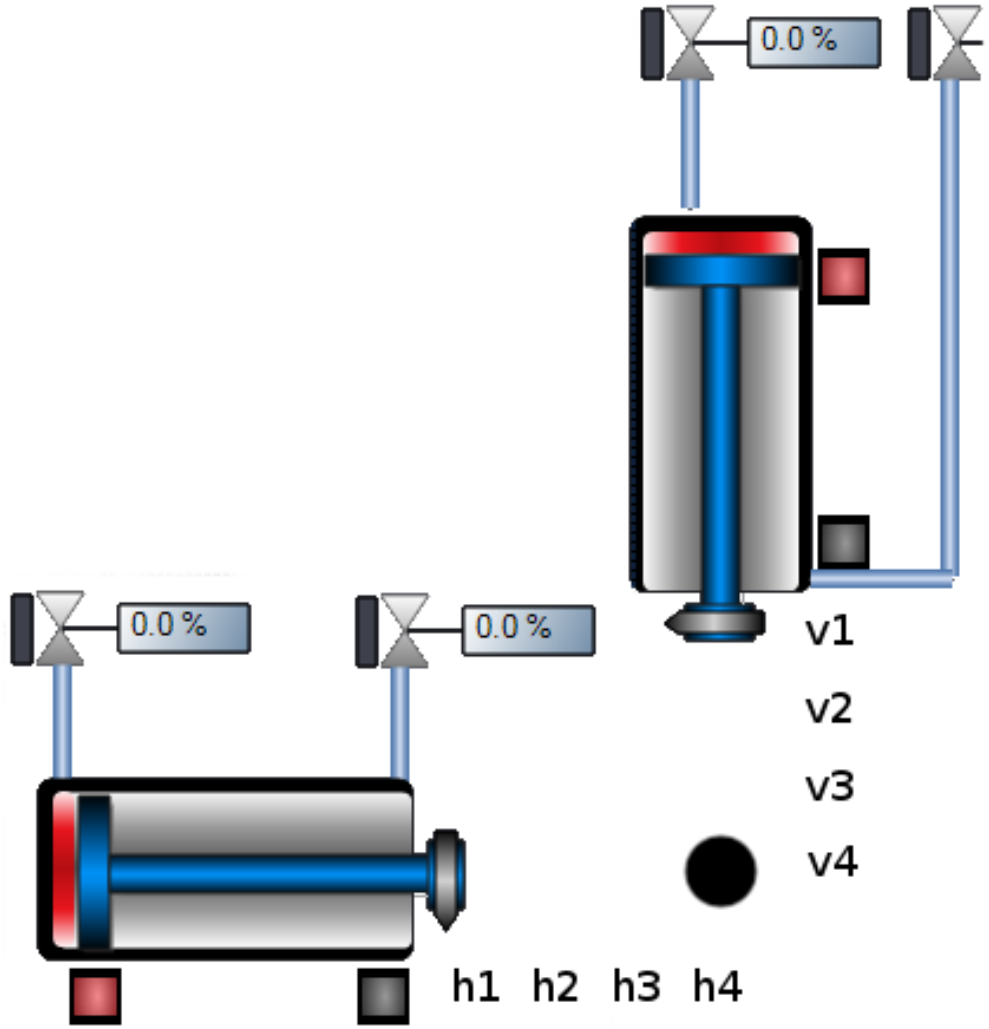
Graph theory approach

- If f is just a Boolean formula, then it is trivial to check whether it holds in a desired state
- **AX** f holds in s, if we previously found that f holds for each in all its successors
- Similar (sometimes more complex) ideas for other operators
- We won't go into details

Symbolic approach

- Implicit representation of states via Boolean formulae
- Allows partial mitigation of the **state explosion problem**
- The symbolic approach will be examined in more detail later in the course

CTL verification: two cylinders



- Atomic propositions: $h_1..h_4, v_1..v_4, w$

Quiz: specify the following properties in CTL:

- If a cylinder is fully extended, then there is no workpiece
- Cylinders do not collide
- When a workpiece appears, it must be eventually pushed away
- Cylinders iterate

Quiz answers

Quiz answers

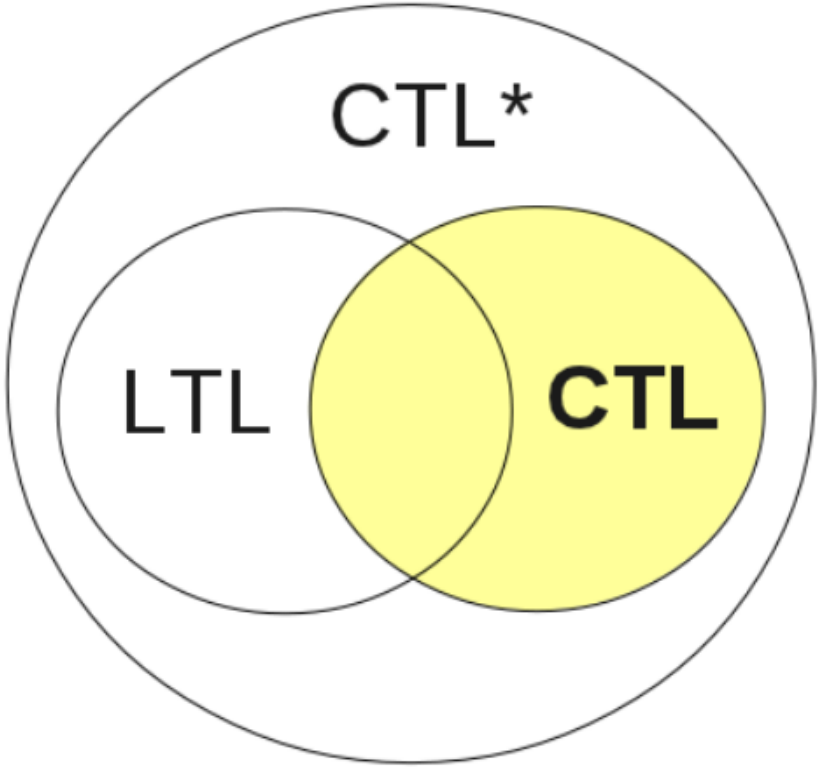
- If a cylinder is fully extended, then there is no workpiece: $\mathbf{AG}(h_4 \vee v_4 \rightarrow \neg w)$
- Cylinders do not collide: $\mathbf{AG}\,\neg(h_4 \wedge v_4)$
- When a workpiece appears, it must be eventually pushed away: $\mathbf{AG}(w \rightarrow \mathbf{AF}\,\neg w)$
- Cylinders iterate: ???

Common specification types ("patterns")

Name	LTL	CTL
Generality / Invariance	$\mathbf{G}\, f$	$\mathbf{AG}\, f$
Bounded response	$\mathbf{G}(p \to \mathbf{X}^n\, q)$	$\mathbf{AG}(p \to (\mathbf{AX})^n q)$
Unbounded response	$\mathbf{G}(p \to \mathbf{F}\, q)$	$\mathbf{AG}(p \to \mathbf{AF}\, q)$
Infinitely often	$\mathbf{G}\,\mathbf{F}\, p$	$\mathbf{AG}\,\mathbf{AF}\, p$

There are properties which cannot be expressed in both LTL and CTL

- **G** p/ **AG** p, **F** p/ **AF** p, **G F** p/ **AG AF** p – both LTL and CTL
- **EF** p – only CTL, but there is a workaround to check it in LTL!
- **F G** p – only LTL
- **AG EF** p – only CTL
- CTL* is a larger logic which allows combining quantified and unquantified temporal operators



Tool support

- UPPAAL: a subset of CTL, some LTL properties can be specified with automata
- NuSMV: both CTL and LTL
- SPIN: LTL

Literature

1. Baier, C., & Katoen, J. P. (2008). Principles of model checking. MIT press.
2. Clarke, E. M., Grumberg, O., & Peled, D. (1999). Model checking. MIT press.
3. Schneider, K. (2013). Verification of reactive systems: formal methods and algorithms. Springer Science & Business Media.