First Inner Olympiad 2019-2020

1. Does there exist a continuous function $f:\mathbb{R}\to\mathbb{R}$, which plot intersects each non-vertical line infinite number of times?

2. Calculate $\int\limits_0^{2008} x(x-4)(x-8)\dots(x-2008)\,\mathrm{d}x$.

3. $f(x)$ is continuous and differentiable on $[0,1]$ with $f(1)=f(0)+1$. Prove that $\int\limits_0^1 (f'(x))^2\,\mathrm{d}x \geq 1$.

4. Solve $y'(t)=y^2(x)\left(1+\int\limits_\pi^x \frac{\mathrm{d}t}{y(t)}\right)$ with $y(\pi)=1$.

5. Find the gcd of $\{2^{13}-2, 3^{13}-3,\dots,n^{13}-n\}$.

6. Let $a_1,a_2,\dots,a_n$ and $b_1,b_2,\dots,b_n$ be nonnegative real numbers. Show that $(a_1a_2\cdots a_n)^{\frac{1}{n}}+(b_1b_2\cdots b_n)^{\frac{1}{n}} \leq ((a_1+b_1)(a_2+b_2)\cdots(a_n+b_n))^{\frac{1}{n}}$.

7. Let $A\in M_n(\mathbb{C})$, $A^TA=I_n$ and n is odd. Prove that $\det(A^2-I_n)=0$.

8. Let x be a real number. Let $a_{i,0}=\frac{x}{2^i}$ and $a_{i,j+1}=a_{i,j}^2+2a_{i,j}$. Find $\lim\limits_{n\to\infty} a_{n,n}$.

9. Let all the roots of the polynomial $P(x)=x^n+a_2x^{n-2}+\ldots+a_{n-1}x+a_n$ be reals. Prove that $a_2\leq 0$.

10. Let $f:[0,1]\to[0,1]$ be the continuous function such that $\int\limits_0^{f(x)} f(y)\,\mathrm{d}y = f(f(x))$ for any $x\in[0,1]$. Find $f(f([0,1]))$.

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