Theorem (Stolz-Cezaro). *Let $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$ be two sequences of real numbers. Assume that b_n is strictly monotone and divergent sequence and the following limit exists* $\lim\limits_{n\to\infty} \frac{a_{n+1}-a_n}{b_{n+1}-b_n} = \ell$. *Then,* $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = \ell$

1. Evaluate $\lim\limits_{n\to\infty} e^{\frac{1}{n+1}+\ldots+\frac{1}{2n}}$.

2. Find the limit $\lim\limits_{n\to\infty} \cos \frac{2\pi e n!}{3}$.

3. Let $a_1 = a$, $a_2 = b$ and $a_n = \sqrt{a_{n-1}a_{n-2}}$. Find the limit $\lim\limits_{n\to\infty} a_n$.

4. Find the limit $\lim\limits_{n\to\infty} \frac{2^n}{a^{2^n}+1}$ with $a > 1$.

5. Find the limit $\lim\limits_{n\to\infty} \frac{1^p+2^p+\ldots+n^p}{n^{p+1}}$.

6. Let (x_n) and (y_n) be two sequences of reals such that the sequences $x_n - y_n$ and $x_n^3 + y_n^3$ have zero as a limit. Prove that $\lim\limits_{n\to\infty} x_n = \lim\limits_{n\to\infty} y_n = 0$

7. Let $(a_n)_{n\geq 1}$ be a sequence of positive real numbers so that $a_1 > 2$ and $a_n = \sqrt{2 + a_{n-1}}$ for all $n \geq 2$. Calculate the following limit $\lim\limits_{n\to\infty} (3 - a_n)^{4^n}$.

8. Let $x_{n+1} = \sin x_n$ and $x_0 \in (0, \pi)$. Find $\lim\limits_{n\to\infty} \sqrt{n} \cdot x_n$.

9. Let $(x_1, y_1) = (0.8, 0.6)$, $x_{n+1} = x_n \cos y_n - y_n \sin y_n$ and $y_{n+1} = x_n \sin y_n + y_n \cos y_n$. Find $\lim\limits_{n\to\infty} x_n$ and $\lim\limits_{n\to\infty} y_n$.

10. Given sequence (a_n) with $\lim\limits_{n\to\infty} a_n \left(\sum\limits_{i=1}^{n} a_i^2\right) = 1$. Prove that $\lim\limits_{n\to\infty} (3n)^{\frac{1}{3}} a_n = 1$.

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