Theorem (Cauchy's functional equation)**.** *If f is continuous and $f(x+y) = f(x)+f(y)$ then $f(x) = cx$.*

1. Find all continuous functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that $f(xy) = xf(y) + yf(x) - 2xy$.
2. Find all continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(x)f(y) = f(\sqrt{x^2 + y^2})$.
3. Find all continuous functions f such that $f(x + y) = f(x) + f(y) + xy \cdot (x + y - 1)$.
4. Prove that there do not exist functions $f, g : \mathbb{R} \to \mathbb{R}$ such that $f(g(x)) = x^{2018}$ and $g(f(x)) = x^{2019}$.
5. Find all continuous functions $f : [0, 1] \to \mathbb{R}$ such that for all $x \in [0, 1]$, $f\left(\frac{x}{2}\right) + f\left(\frac{x+1}{2}\right) = 3f(x)$.
6. Find all polynomials such that $P(x)^2 = P(x^2)$.
7. Find all functions $f : S_n \to S_n$ such that $sf(s)^3f(t)^2 = tf(t)^3f(s)^2$ for all permutations $s, t \in S_n$.
8. Find all differentiable functions $f : (0, +\infty) \to \mathbb{R}$ such that $f(b) - f(a) = (b - a)f'(\sqrt{ab})$ for all $a, b > 0$.
9. Find all integrable functions $f : [0, 1] \to \mathbb{R}$ such that $\int\limits_0^x f(t)\,\mathrm{d}t = f(x)^{2015} + f(x)$ for all $x \in [0, 1]$.
10. Let $c > 0$ be an arbitrary constant. Find all continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that the following equation is satisfied: $f(x) = f(x^2 + c)$ for all x.

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