1. Look at english wikipedia for Liouville numbers.

2. We will prove that it is a Liouville number, and, thus, trancendental.

Let us fix n. We choose $q = (n!)!$ and $p = q \sum_{k=1}^{n} \frac{1}{(k!)!}$. Then $0 < \left| \sum_{k=1}^{\infty} \frac{1}{(k!)!} - \frac{p}{q} \right| = \left| \sum_{k=n+1}^{\infty} \frac{1}{(k!)!} \right| < \left| \sum_{k=1}^{\infty} \frac{1}{(n!)!^n 2^k} \right| = \frac{1}{(n!)!^n}$.

3-. Look at the complementary material. Yimin Ge, Elementary Properties of Cyclotomic Polynomials.

4. Complementary material. Theorem 2.

5. Complementary material. Lemma 2 and Corollary 2.

6. Complementary material. Theorem 3.

7. Complementary material. Lemma 3.

8. Complementary material. Lemma 4 and Corollary 4.

9. Complementary material. Theorem 4.

10. Complementary material. Theorem 5.

11. Complementary material. Theorem 6.

12. Let $p_n(x) = x^{2^n} + x^{2^{n-1}} + 1$ and $q_n(x) = x^{2^n} - x^{2^{n-1}} + 1$. Then, $p_n(x) = p_1(x)q_1(x) \cdots q_{n-1}(x)$.

It is clear that $p_1(x) = x^2 + x + 1$ is irreducible.

Also, $\Phi_{3 \cdot 2^n}(x) = \frac{\Phi_{2^n}(x^3)}{\Phi_{2^n}(x)} = \frac{x^{3 \cdot 2^{n-1}}+1}{x^{2^{n-1}}+1} = x^{2 \cdot 2^{n-1}} - x^{2^{n-1}} + 1 = q_n(x)$ is irreducible.